


^{le}
Explanation of the characters
made use of in the following
parts of the Work, page 53

Multiply the square of the
polygon by the number standing
opposite to its name in the
following table and the product
will be the area ^{page} 65



William. L. C. Penington

September 15. 1818.



INTRODUCTION
TO
MENSURATION
AND
PRACTICAL GEOMETRY.

BY JOHN BONNYCASTLE,
OF THE ROYAL MILITARY ACADEMY, WOOLWICH.

TO WHICH IS ADDED,
AN APPENDIX,
CONTAINING
A CONCISE SYSTEM OF GAUGING.

THE FIRST AMERICAN, FROM THE TENTH
LONDON EDITION,
REVISED AND CORRECTED.

PHILADELPHIA,
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1812.

1127
William. L. Conrad.

DISTRICT OF PENNSYLVANIA, TO WIT:

(L. S.) BE IT REMEMBERED, That on the eighth day of June, in the thirty-sixth year of the Independence of the United States of America, A. D. 1812,

KIMBER AND CONRAD,
of the said district, have deposited in this office the title of a book, the right whereof they claim as proprietors, in the words following, to wit:

An Introduction to Mensuration and Practical Geometry.
By John Bonnycastle, of the Royal Military Academy, Woolwich. To which is added, an Appendix, containing a Concise System of Gauging. The first American from the tenth London edition, revised and corrected.

In conformity to the act of the Congress of the United States, intituled, "An act for the encouragement of learning, by securing the copies of maps, charts, and books, to the authors and proprietors of such copies during the times therein mentioned." And also to the act, entitled, "An act supplementary to an act, entitled, "An act for the encouragement of learning, by securing the copies of maps, charts, and books to the authors and proprietors of such copies during the times therein mentioned," and extending the benefits thereof to the arts of designing, engraving, and etching historical and other prints."

D. CALDWELL,
Clerk of the District of Pennsylvania.

PREFACE

TO THE LONDON EDITION.

THE ART OF MEASURING, like all other useful inventions, appears to have been the offspring of want and necessity; and to have had its origin in those remote ages of antiquity, which are far beyond the reach of credible and authentic history. Egypt, the fruitful mother of almost all the liberal sciences, is imagined likewise to have given birth to GEOMETRY or MENSURATION; it being to the inundations of the Nile that we are said to be indebted for this most perfect and delightful branch of human knowledge.

After the overflowings of the river had deluged the country, and all artificial boundaries and landmarks were destroyed, there could have been no other method of ascertaining individual property, than by a previous knowledge of its figure and dimensions. From this circumstance, it appears highly probable, that Geometry was first known and cultivated by the ancient Egyptians; as being the only science which could administer to their wants, and furnish them with the assistance they required. The name itself signifies properly the *art of measuring the earth*; which serves still further to confirm this opinion; especially as it is well known that many of the ancient mathematicians applied their geometrical knowledge entirely to that purpose; and that even the Elements of Euclid, as they now stand, are only the theory from whence we

obtain the rules and precepts of our present more mechanical practice.

But to trace the sciences to their first rude beginnings is a matter only of learned curiosity, which could afford but little gratification to readers in general. It is of much more consequence to the rising generation to be informed that, in their present improved state, they are exceedingly useful and important. And in this respect, the art I have undertaken to elucidate is inferior to none. arithmetic only excepted. Its use in most of the different branches of the mathematics is so general and extensive, that it may justly be considered as the mother and mistress of all the rest, and the source from whence were derived the various properties and principles to which they owe their existence.

As a testimony of this superior excellence, I need only mention a few of those who have studied and improved it; in which illustrious catalogue we have the names of Euclid, Archimedes, Thales, Anaxagoras, Pythagoras, Plato, Apollonius, Philo, and Ptolemy, among the ancients; and Huygens, Wallis, Gregory, Halley, the Bernouillies, Euler, Leibnitz, and Newton, among the moderns; all of whom applied themselves to particular parts of it, and greatly enlarged and improved the subject. To the latter especially we are indebted for many valuable discoveries in the higher branches of the art; which have not only enhanced its dignity and importance, but rendered the practical application of it more general and extensive.

The degree of estimation in which the art was held by these and other eminent characters, will, in general, it is apprehended, be thought a sufficient encomium on its merits. But, for the sake of young people, and those of a confined education, it may not be amiss to give a few more instances of its advan-

PREFACE.

v

age, and show that its importance in trade and business is not inferior to its dignity as a science. Artificers of almost all denominations are indebted to this invention for the establishment of their several occupations, and the perfection and value of their workmanship. Without its assistance all the great and noble works of art would have been imperfect and useless. By this means the architect lays down his plan, and erects his edifice; bridges are built over large rivers; ships are constructed; and property of all kinds is accurately measured, and justly estimated. In short, most of the elegances and conveniencies of life owe their existence to this art, and will be multiplied in proportion as it is well understood, and properly practised.

From this view of the subject, it is hardly to be accounted for, that, in a commercial nation, like our own, an art of such general application should have been so greatly neglected. Mechanics of all kinds, it is well known, are but ill acquainted with mathematics; and those who have been the best qualified to afford them any assistance have thought it beneath their attention. Till within a few years past there could not be found a regular treatise upon this subject in the English language. Some particular branches, it is true, had been greatly cultivated and improved; but these were only to be found in their miscellaneous state, interspersed through a number of large volumes, in the possession of but few, and in a form and language totally unintelligible to those for whom they were more immediately necessary.

Dr. Hutton was the first person, in this country, who undertook to collect these scattered fragments, and to treat of the subject in a scientific, methodical manner. A small treatise by Hawney, and some others of little note, had indeed been long in the

hands of the public; but these were extremely defective, both in matter and method; neither the principles nor practice of the art being properly or clearly explained. Before the publication of the treatise above-mentioned, Mr. Robertson's may be considered as the only book of any value that could be consulted, either by the artizan or mathematician; and had he given the theory as well as the practice of the art, and divested his rules and examples of their algebraical form, there would have been no want of any other elementary treatise.

To these two writers I am greatly indebted for many things in the following pages, and am ready to acknowledge, that I have used an unreserved freedom in selecting from their works, wherever I found them to answer my purpose. To Dr. Hutton I am particularly obliged, and am so far from desiring to supersede the use of his performance by this publication, that I only wish it to be thought a useful introduction to it. His treatise is excellent in its kind; and had it been as well calculated for the use of the uninformed artist as it is for the mathematician, the following compendium had certainly never been published.

The method I have observed, in composing this work, is that which was used in the "*Scholar's Guide to Arithmetic*;" and, as my object has been to facilitate the acquirement of the same kind of useful knowledge, I am not without hopes of its being received with equal candour and approbation.

In school-books, and those designed for the use of learners, it has always appeared to me, that plain and concise rules, with proper exercises, are entirely sufficient for the purpose. In science, as well as in morals, example will ever enforce and illustrate precept; for this reason, an operation, wrought out at length, will be found of more service to begin-

more than all the tedious directions and observations that can possibly be given them. From constant experience I have been confirmed in this idea; and it is in pursuance of it that I have formed the plan of this publication. I have not been ambitious of adding much new matter to the subject; but only to arrange and methodize it in a manner more easy and rational than had been done before.

Some of the most difficult rules relating to the surfaces of solids, &c. could not be conveniently given, but by means of algebraical theorems; and as this was foreign to my purpose, I have not scrupled to omit them; being well persuaded that what is done upon that head will be fully sufficient to answer most practical purposes. In the Practical Geometry likewise, which is prefixed to this treatise, such problems only are introduced as were known to be most intimately connected with the subject. And as this part of the work is a proper and necessary introduction to the rest, I have spared no pains in making it as clear and intelligible as possible.

Upon the whole, I have endeavoured to consult the wants of the learner more than those of the man of science. And if I have succeeded in this respect, my purpose is answered. I have not sought for reputation as a mathematician, but only to be useful as a tutor.

N. B. The favourable reception this work has met with has induced me, in this edition, to make such alterations and additions as have since occurred to me, and which are such as I hope will render it still more acceptable to the public.

*Royal Academy, Woolwich,
July 14, 1807.*



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TABLES

OF THE

DIFFERENT MEASURES USED IN THIS WORK

Linear Measures.

Square Measures.

12 inches make 1 foot.	144 inches make 1 foot.
3 feet 1 yard.	9 feet 1 yard.
6 feet 1 fathom.	56 feet 1 fathom.
16½ feet, or } { 1 pole,	272½ feet, } { 1 pole,
5½ yards, } { or rod.	or 30½ yds. } { or rod.
40 poles 1 furlong.	1600 poles 1 furlong.
8 furlongs 1 mile.	64 furlongs 1 mile.

Note. The chain made use of in measuring land, commonly called Gunter's chain, is 4 poles, or 22 yards in length, and consists of 100 equal links, each link being $\frac{22}{100}$ of a yard, or 7½ inches long.

An acre of land is also equal to 160 square chains; that is, 10 chains in length, and 1 in breadth; or it is 4840 square yards, or 160 square poles, or 100,000 square links.

Note also, that in Land Measure,

And in Cubic Measure,

40 perches, or } make 1 rood.	1728 inches make 1 foot.
square poles } make 1 rood.	27 feet 1 yard.
4 roods 1 acre.	166⅔ yards 1 pole.

PRACTICAL GEOMETRY.

DEFINITIONS.

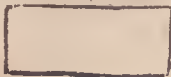
1. *GEOMETRY* is that science which treats of the descriptions and properties of magnitudes in general.

2. A *point* is that which has no parts, or dimensions.

3. A *line* is length without breadth; and its bounds or extremes are points.

4. A *right line* is that which lies evenly between its extreme points.

5. A *superficies* is that which has length and breadth only; and its bounds or extremes are lines.



6. A *plane superficies* is that in which any two points being taken, the right line that joins them lies wholly in that superficies.

7. A *solid* is that which has length, breadth, and thickness; and its bounds, or extremes, are superficies.



8. A *plane rectilinear angle* is the inclination or opening of two right lines which meet in a point.



9. One line is said to be *perpendicular* to another, when it makes the angles on both sides of it equal to each other.



10. A *right angle* is that which is formed by two lines that are perpendicular to each other*.



* Any angle differing from a right one, is, by some writers, called an *oblique angle*.

11. An *acute angle* is that which is less than a right angle.



12. An *obtuse angle* is that which is greater than a right angle.



13. A *circle* is a plane figure, formed by the revolution of a right line about one of its extremities, which remains fixed*.



14. The *centre* of a circle is the point about which it is described; and the *circumference* is the line or boundary by which it is contained.

* N. B. The circumference itself, for the sake of conciseness, is sometimes called a circle.

15. The *radius* of a circle is a right line drawn from the centre to the circumference.



16. The *diameter* of a circle is a right line passing through the centre, and terminated both ways by the circumference.



17. An *arc* of a circle is any part of its periphery or circumference.



18. A *chord* is a right line joining the extremities of an arc.



N. B. A *semicircle* is half the circle, and a *quadrant* the quarter of it.

19. All plane figures bounded by three right lines are called *triangles*.

20. An *equilateral triangle* is that whose three sides are all equal.



21. An *isosceles triangle* is that which has only two of its sides equal.



22. A *scalene triangle* is that which has all its three sides unequal.



23. A *right-angled triangle* is that which has one right angle*.



* Any triangle differing from a right-angled one, is called an *oblique-angled triangle*.

24. An *obtuse-angled triangle* is that which has one obtuse angle.



25. An *acute-angled triangle* is that which has all its angles acute.



26. All plane figures, bounded by four right lines, are called *quadrangles*, or *quadrilaterals*.

27. A *square* is a quadrilateral, whose sides are all equal, and its angles all right angles.

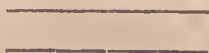


28. A *rhombus* is a quadrilateral, whose sides are all equal, but its angles not right angles*.

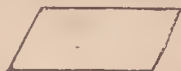


* This figure, by working mechanics, is sometimes called a lozenge.

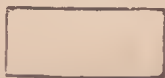
29. *Parallel lines* are such as are in the same plane, and which, being produced ever so far both ways, do not meet.



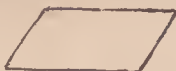
30. A *parallelogram* is a quadrilateral whose opposite sides are parallel.



31. A *rectangle* is a parallelogram whose angles are all right angles.



32. A *rhomboid* is a parallelogram whose angles are not right angles.



33. All other four-sided figures, besides these are called *trapeziums*.

34. A right line joining any two opposite angles of a four-sided figure is called the *diagonal*.



35. All plane figures contained under more than four sides are called *polygons*.

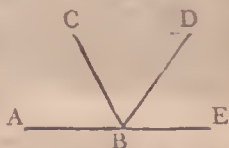
36. Polygons having five sides, are called *pentagons*; those of six sides, *hexagons*; those of seven, *heptagons*; and so on.

37. A *regular polygon* is that whose angles, as well as sides, are all equal.

38. The *base* of any figure is the side on which it is supposed to stand; and the *altitude* is the perpendicular falling upon it from the opposite angle.

39. In a right-angled triangle the side opposite to the right angle is called the *hypotenuse*; and the other two sides are called *legs*.

40. An angle is usually denoted by three letters, the one which stands at the angular point being always to be read in the middle.



41. The circumference of every circle is supposed to be divided into 360 equal parts, called *degrees*; each degree into 60 equal parts called *minutes*; and so on.

42. The *measure* of any right-lined angle is an arc of a circle contained between the two lines which form that angle, the angular point being the centre.

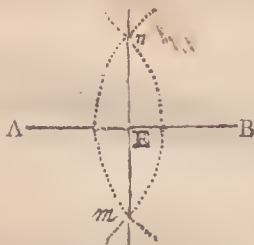


* This and the following definition are used in Practical Geometry.

Note. The angle is estimated by the number of degrees contained in the measuring arc; whence a right angle is an angle of 90 degrees, being measured by $\frac{1}{4}$ of the circumference.

PROBLEM I*.

To divide a given line AB into two equal parts



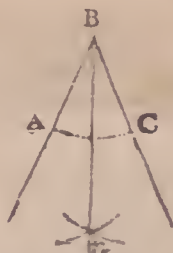
1. From the points A and B, as centres, with any distance greater than half AB, describe arcs cutting each other in *n* and *m*.

2. Through these points draw the line *nEm*, and the point E, where it cuts AB, will be the middle of the line required.

PROBLEM II.

To divide a given angle ABC into two equal parts.

* The demonstrations of most of these Problems may be found in Euclid's Elements.



1. From the point B, with any radius, describe the arc AC.

2. And from A, C, with the same, or any other radius, describe arcs cutting each other in n .

3. Through the point n draw the line BD , and it will bisect the angle ABC , as was required.

PROBLEM III.

From a given point C, in a given right line AB, to erect a perpendicular.

CASE I. *When the point is near the middle of the line.*



1. On each side of the point C take any two equal distances Cn , Cm .

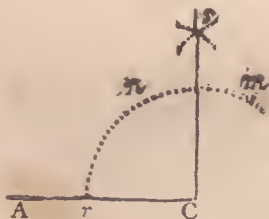
2. From n and m , with any radius greater than Cn or Cm , describe arcs cutting each other in s .
3. Through the point s , draw the line sC , and it will be the perpendicular required.

CASE II. *When the point is at, or near, the end of the line.*



1. Take any point o , not in AB , and with the radius or distance oC , describe the arc mCn , cutting AB in m and C .
2. Through the centre o , and the point m , draw the line msn , cutting the arc mCn in n .
3. From the point n , draw the line nC , and it will be the perpendicular required.

Another method.



1. From the point C , with any radius, describe the arc rn , cutting the line AC in r .

2. With the same radius, and r as a centre, cross the arc in n ; and from n , with the same radius, cross it in m .

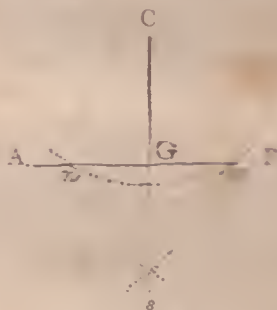
3. From the points n and m , with the same, or any other radius, describe arcs cutting each other in s .

4. Through the point s , draw the line SC , and it will be the perpendicular required*.

PROBLEM III.

From a given point C, one of a given line AB, to let fall a perpendicular.

CASE I. *When the point is exactly equidistant to the middle of the line.*



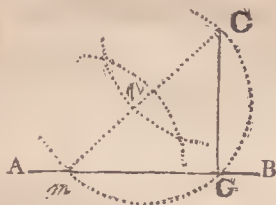
1. From the point C, with any radius, describe the arc nm , cutting AB in n and m .

2. From the points n , m , with the same, or any other radius, describe two arcs cutting each other in s .

* Another method of raising a perpendicular from a point in a given line may be seen in problem XXX.

3. Through the points C, s , draw the line CGs , and CG will be the perpendicular required.

CASE II. *When the point is nearly opposite to the end of the line.*



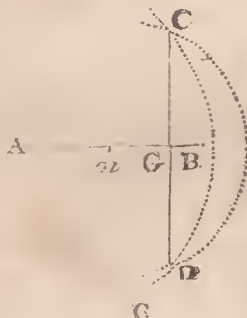
1. To any point m , in the line AB , draw the line Cm .

2. Bisect the line Cm , or divide it into two equal parts, in the point n .

3. From n , with the radius nm , or nC , describe the arc CGm , cutting AB in G .

4. Through the point C , draw the line CG , and it will be the perpendicular required.

Another Method.



1. From A , or any other point in AB , with the radius AC , describe the arc CD .

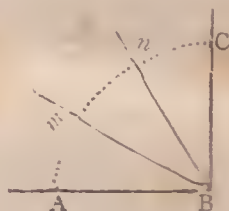
2. And from any other point n . in AB . with the radius nC , describe another arc cutting the former in C , D .

3. Through the points C , D , draw the line CGD , and CG will be the perpendicular required.

N. B. Perpendiculars may be more easily raised, and let fall, in practice, by means of a square, or other proper instrument.

PROBLEM V.

To trisect, or divide a right angle ABC into three equal parts.



1. From the point B , with any radius BA , describe the arc AC , cutting the legs BA , BC , in A , C .

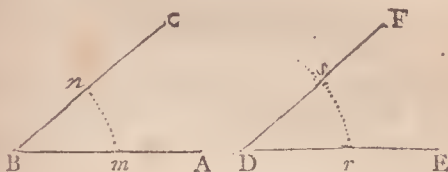
2. And from the point A , with the radius AB or BC , cross the arc AC in n .

3. Also with the same radius, from the point C , cross it in m .

4. Through the points m , n , draw the lines Bn , Bm , and they will trisect the angle as was required.

PROBLEM VI.

At a given point D to make an angle equal to a given angle ABC.



1. From the point B, with any radius, describe the arc nm , cutting the legs BA, BC, in the points m , n .

2. Draw the line DE, and from the point D, with the same radius as before, describe the arc rs .

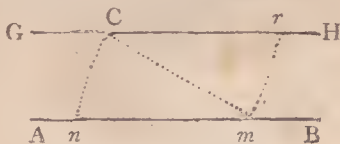
3. Take the distance mn , on the former arc, and apply it to the arc rs , from r to s .

4. Through the points D, s , draw the line DF, and the angle EDF will be equal to the angle ABC, as was required.

PROBLEM VII.

To draw a line parallel to a given line AB.

CASE I. *When the parallel line is to pass through a given point C.*



1. To AB , from the point C , draw any right line Cm .

2. From the point m , with the radius mC , describe the arc Cn , cutting AB in n .

3. And with the same radius, from the point C , describe the arc mr .

4. Take the distance Cn , and apply it to the arc mr , from m to r .

5. Through the points C, r , draw the line $GCrH$, and it will be parallel to AB , as was required.

CASE II. *When the perpendicular distance is greater than the distance from AB .*



1. From any two points r, s , in the line AB , with a radius equal to the given distance, describe the arcs n, m .

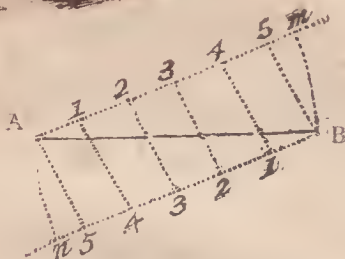
2. Draw the line DG , to touch these arcs without cutting them, and it will be parallel to AB , as was required.

N. B. The former case of this problem, as well as several other operations in Practical Geometry, may be more easily effected by means of the parallel ruler*.

* This ruler may be had of all sizes, but is usually put into a portable case, with a drawing-pen, scale, compasses and other useful instruments.

PROBLEM VIII.

To divide a given line AB into any proposed number of equal parts.

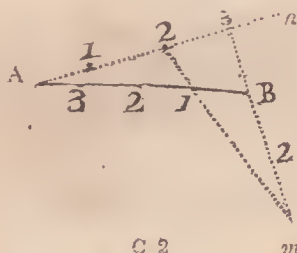


1. From one end of the line A, draw Am, making any angle with AB; and from B, the other end, draw Bn, making an equal angle ABn.

2. In each of the lines Am, Bn, beginning at A and B, set off as many equal parts, of any length, as AB is to be divided into.

3. Join the points A5, 1 4, 2 3, &c. and AB will be divided as was required.

Note. Bn may be drawn parallel to Am, by means of a parallel ruler.

Another Method.

1. From the point A draw any line An , and set on it as many equal parts, wanting one, as AB is to be divided into.

2. Through the points 3. B , draw the line $3Bm$, and take upon it the same number of parts, each equal to $3B$.

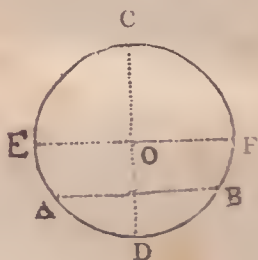
3. From the point m , in $3Bm$, draw the line $m12$, cutting AB in 1.

4. Upon AB , take the parts 1 2. 2 3. and 3 4. each equal to $B1$, and the line will be divided, as was required.

Note. It will be convenient in practice to draw An so that BAn shall be a small angle.

PROBLEM IX.

To find the centre of a given circle, or one already described.*



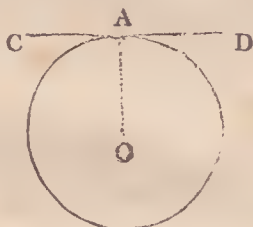
1. Draw any chord AB , and bisect it with the perpendicular CD .

2. Bisect CD , in like manner, with the chord EF , and their intersection O , will be the centre required

PROBLEM X.

To draw a tangent to a given circle, that shall pass through a given point A.

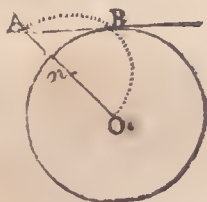
CASE I. *When the point A is in the circumference of the circle.*



1. From the given point A, to the centre of the circle, draw the radius OA.

2. Through the point A, draw CD perpendicular to OA, and it will be the tangent required.

CASE II. *When the point A is without the circle.*



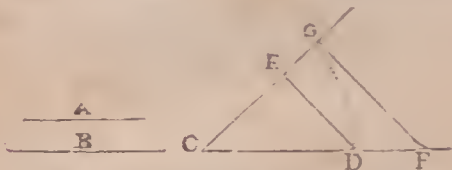
1. To the point A, from the centre O, draw the line OA, and bisect it in n.

2. From the point n , with the radius nA , or nO , describe the semicircle ABO , cutting the given circle in B .

3. Through the points A, B , draw the line AB , and it will be the tangent required.

PROBLEM XI

To two given right lines A, B , to find a third proportional.



1. From the point C draw two right lines, making any angle FCG .

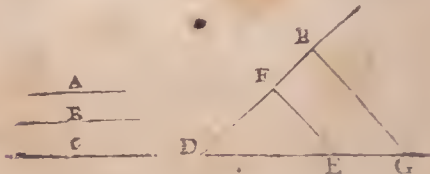
2. In these lines take CE equal to the first term A , and CG, CD , each equal to the second term B .

3. Join ED , and draw GF parallel to it; and CF will be the third proportional required.

That is, $CE (A) : CG (B) :: CD (B) : CF$.

PROBLEM XII.

To three given right lines A, B, C , to find a fourth proportional.



1. From the point D draw two right lines, making any angle GDH.

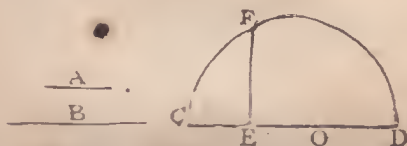
2. In these lines take DF equal to the first term A, DE equal to the second B, and DH equal to the third C.

3. Join FE, and draw HG parallel to it, and DG will be the fourth proportional required.

That is, $DF (A) : DE (B) :: DH (C) : DG$.

PROBLEM XIII.

Between two given right lines A, B, to find a mean proportional.



1. Draw any right line, in which take CE equal to A, and ED equal to B.

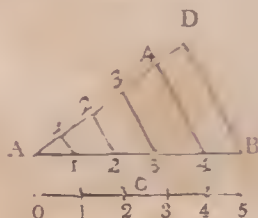
2. Bisect CD in O, and with OD or OC, as radius, describe the semicircle CFD.

3. From the point E draw EF perpendicular to CD, and it will be the mean proportional required.

That is, $CE (A) : EF :: EF : ED (B)$.

PROBLEM XIV.

To divide a given line AB in the same proportion that another given line C is divided.*



1. From the point A draw AD equal to the given line C, and making any angle with AB.

2. To AD apply the several divisions of C, and join DB.

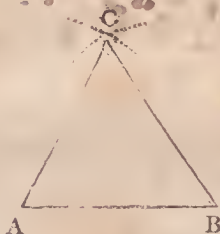
3. Draw the lines 4 4, 3 3, &c. each parallel to DB, and the line AB will be divided as required.

That is, the parts A 1, 1 2, 2 3, 3 4, 4 B, on the line AB, will be proportioned to the parts 0 1, 1 2, 2 3, 3 4, 4 5, on the line C.

PROBLEM XV.

Upon a given right line AB, to make an equilateral triangle.

* The case of this problem which most frequently occurs, is that in which the given line is required to be divided into two parts that shall have a given ratio, which may be done in nearly the same manner as above.



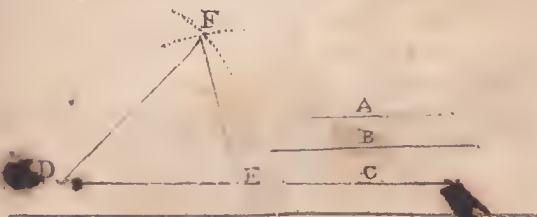
1. From the points A and B, with a radius equal to AB, describe arcs cutting in C.

2. Draw the lines AC, BC, and the figure ACB will be the triangle required.

Note. An isosceles triangle may be formed in the same manner, by taking any distance for radius.

PROBLEM XVI.

To make a triangle whose three sides shall be respectively equal to three given lines, A, B, C.*

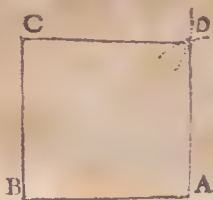


* The three given lines must be each of such a length that any two of them taken together shall be greater than the third.

1. Draw a line DE, equal to one of the given lines C.
2. On the point D, with a radius equal to B, describe an arc.
3. And on the point E, with a radius equal to A, describe another arc, cutting the former in F.
4. Draw the lines DF, EF, and DFE will be the triangle required.

PROBLEM XVII.

Upon a given line AB to describe a square

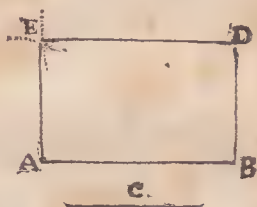


1. From the point B, draw BC perpendicular, and equal to AB.
2. On A and C, with the radius AB, describe two arcs cutting each other in D.
3. Draw the lines AD, CD, and the figure ABCD will be the square required.

Note. A rhombus may be made on the given line AB in exactly the same manner, if BC be drawn with the proper obliquity, instead of perpendicularly.

PROBLEM XVIII.

To describe a rectangle, whose length and breadth shall be equal to two given lines AB and C.



1. At the point B, in the given line AB, erect the perpendicular BD, and make it equal to C.

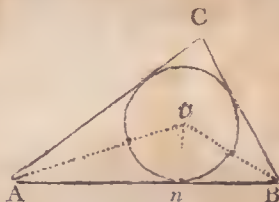
2. From the points D, A, with the radii AB and C, describe two arcs cutting each other in E.

3. Join EA and ED, and ABDE will be the rectangle required.

Note. A parallelogram may be described in nearly the same manner.

PROBLEM XIX.

To inscribe a circle in a given triangle ABC

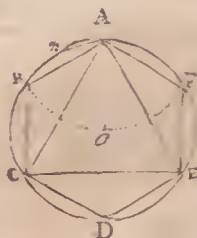


1. Bisect the angles A and B with the lines AO and BO.

2. From the point of intersection O let fall the perpendicular On, and it will be the radius of the circle required.

PROBLEM XX.

In a given circle to inscribe an equilateral triangle, an hexagon, or a dodecagon.



For the hexagon.

1. From any point A as a centre, with a distance equal to the radius Ao, describe the arc BoF.

2. Join the points AB, or AF, and either of these lines being carried six times round the circle will form the hexagon required.

That is, the radius of the circle is equal to the side of the hexagon; and the sides of the hexagon divide the circumference of the circle into six equal parts, each containing 60 degrees.

For the equilateral triangle.

1. From the point A, to the second and fourth divisions, or angles of the hexagon, draw the lines AC, AE.

2. Join the points CE, and ACE will be the equilateral triangle required; the arc AC being one third of the circumference, or 120 degrees.

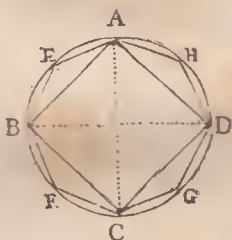
For the dodecagon.

Bisect the arc AB of the hexagon in the point n , and the line An being carried twelve times round the circumference, will form the dodecagon required, the arc An being 30 degrees.

If An be again bisected, a polygon may be formed of 24 sides; and by another bisection, a polygon of 48 sides; and so on.

PROBLEM XXI.

To describe a square, or an octagon, in a given circle.



For the square.

1. Draw the diameters BD and AC, intersecting each other at right angles.
2. Join the points AB, BC, CD, and DA, and ABCD will be the square required.

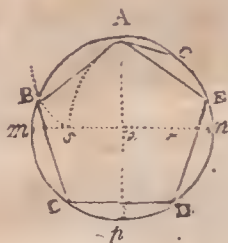
For the octagon.

Bisect the arc AB of the quadrant in the point E, and the line AE being carried eight times round the circumference, will form the octagon.

If the arc AC be again bisected, a polygon may be formed of 16 sides; and by another bisection a polygon of 32 sides; and so on.

PROBLEM XXII*.

To inscribe a pentagon, or decagon, in a given circle.



For the pentagon.

1. Draw the diameters AB and CD at right angles to each other, and bisect the radius OA in r .

2. From the point r , with the distance rA , describe the arc As , and from the point A , with the distance As , describe the arc sB .

3. Join the points A , B , and the line AB being carried five times round the circle, will form the pentagon required.

* Besides the figures here constructed, and those arising from thence by continual bisections, or taking the differences, no other regular polygon can be described from any known method *purely geometrical*.

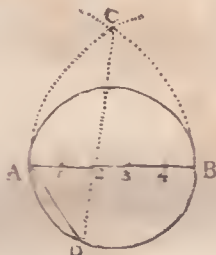
For the decagon.

Bisect the arc AE of the pentagon in *c*, and the line Ac being carried ten times round the circumference will form the decagon required.

If the arc Ac be again bisected, a polygon of 20 sides may be formed; and by another bisection, a polygon of 40 sides; and so on.

PROBLEM XXIII*.

In a given circle to inscribe any regular polygon.



1. Draw the diameter AB, which divide into as many equal parts as the figure has sides.

2. From the points A, B, as centres, with the radius AB, describe arcs crossing each other in C.

3. From the point C, through the second division of the diameters, draw the line CD.

* This construction is the invention of *Renaldinus*, and was first given in his 2d Book *De Resol. &c. Comp. Mathem.* page 367. The rule for polygons, in general, is only an approximation, but holds true in the equilateral triangle and hexagon.

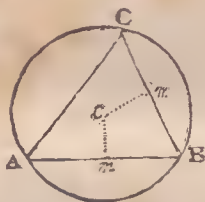
4. Join the points A, D, and the line AD will be the side of the polygon required.

Note. In this construction AD is the side of a pentagon.

Another method, something more accurate, is by erecting a perpendicular from the centre, of such a length that the part without the circle shall be equal to $\frac{3}{4}$ of that within, and drawing a line from its extremity through the second division as before.

PROBLEM XXIV.

About a given triangle ABC to circumscribe a circle.

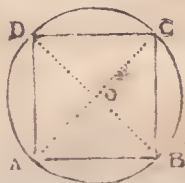


1. Bisect the two sides AB, BC with the perpendicular *mc* and *nc*.

2. From the point of intersection *c*, with the distance *cA*, *cB* or *cC*, describe the circle ACB, and it will be that required.

PROBLEM XXV.

About a given square ABCD to circumscribe a circle.

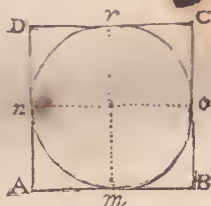


1. Draw the two diagonals AC and BD, intersecting each other in O.

2. From the point O, with the distance OA, OB, OC, or OD, describe the circle ABCD, and it will be that required.

PROBLEM XXVI.

To circumscribe a square about a given circle.



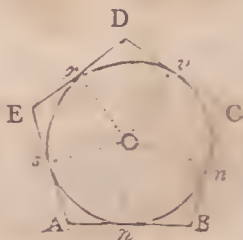
1. Draw any two diameters no and rm at right angles to each other.

2. Through the points m, o, r, n , draw the lines AB, BC, CD, and DA, perpendicular to rm and no , and ABCD will be the square required*.

* If each of the quadrants rm, mn, mo , and or be bisected, and tangents be drawn to those points, the circumscribing figure will be an octagon.

PROBLEM XXVII.

About a given circle to circumscribe a pentagon.



1. Inscribe a pentagon in the circle; or, which is the same thing, find the points n, m, v, r, s , as in Problem XXII.

2. From the centre o , to each of these points, draw the radii on, om, ov, or , and os .

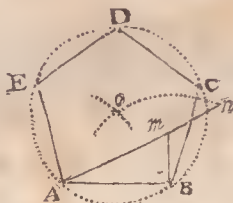
3. Through the points n, m , draw the lines AB, BC perpendicular to on, om ; producing them till they meet each other at B .

4. In the same manner, draw the lines CD, DE, EA , and $ABCDE$ will be the pentagon required.

Note. Any other polygon may be made to circumscribe a circle, by first inscribing a similar one, and then drawing tangents to the circle at the angular points.

PROBLEM XXVIII*.

On a given line AB to make a regular pentagon.



1. Make Bm perpendicular to AB , and equal to one half of it.

2. Draw Am , and produce it till the part mn is equal to Bm .

3 From A and B as centres, with the radius Bn , describe arcs cutting each other in o .

4. And from the point o , with the same radius, or with oA , or oB , describe the circle $ABCDE$.

5. Apply the line AB five times round the circumference of this circle, and it will form the pentagon required.

Note. If tangents be drawn through the angular points A, B, C, D, E , a pentagon circumscribing the circle will be formed; and if the arcs be bisected, a circumscribing decagon may be formed.

* In the former edition of this work, another method of describing a pentagon was given, as first proposed by *Albertus Durer*, in his *Geometry*, p. 55, printed 1532; but as that is only an approximation, and is not more easy in practice than the present one, which is perfectly accurate, it is here omitted.

PROBLEM XXIX.

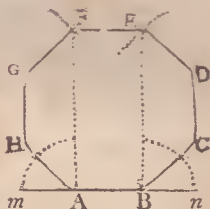
On a given line AB to make a regular hexagon.



1. From the points A, B as centres, with the radius AB, describe arcs cutting each other in *o*.
2. And from the point *o*, with the distance *oA* or *oB*, describe the circle ABCDEF.
3. Apply the line AB six times round the circumference, and it will form the hexagon required*.

PROBLEM XXX.

On a given line AB to form a regular octagon.



* This construction is founded on the principle, that the radius of every circle is equal to the side of its inscribed hexagon, or the chord of 60° .

1. On the extremes of the given line AB erect the indefinite perpendiculars AF and BE .

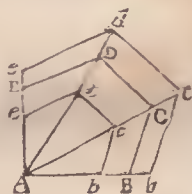
2. Produce AB both ways to m and n , and bisect the angles mAF and nBE with the lines AH and BC .

3. Make AH and BC each equal to AB , and draw HG , CD parallel to AF or BE , and also each equal to AB .

4. From G , D , as centres, with a radius equal to AB , describe arcs crossing AF , BE , in F and E ; and if GF , FE , and ED be drawn, $ABCDEFGH$ will be the octagon required.

PROBLEM XXXI.

To make a figure similar to a given figure $ABCDE$.



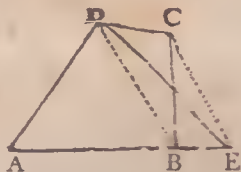
1. Take Ab equal to the side of the figure required, and from the angle A draw the diagonals AC , AD .

2. From the points b , c , d draw bc , cd , de parallel to BC , CD , DE , and $Abcde$ will be similar to $ABCDE$.

The same thing may also be done by making the angles b , c , d , e respectively equal to the angles B , C , D , E .

PROBLEM XXXII.

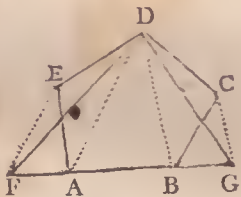
To make a triangle equal to a given trapezium ABCD.



1. Draw the diagonal DB, and make CE parallel to it, meeting the side AB produced in E.
2. Join the points D, E, and ADE will be the triangle required.

PROBLEM XXXIII.

To make a triangle equal to any pentagon ABCDEA.

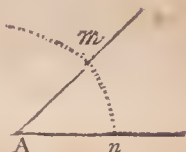
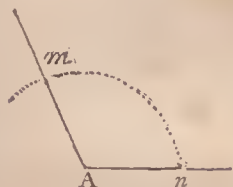


1. Produce the side AB both ways at pleasure.
2. Draw the diagonals DA, DB, and parallel to them the lines EF and CG.
3. Join the points DF, DG, and DFG will be the triangle required.

And in nearly the same manner may any right-lined figure whatever be reduced to a triangle.

PROBLEM XXXIV*.

To make an angle of any proposed number of degrees.



1. Take the first 60 degrees from the scale of chords, and from the point A, with this radius, describe the arc nm .

2. Take the chord of the proposed number of degrees from the same scale, and apply it from n to m .

3. From the point A draw the lines An and Am , and they will form the angle required.

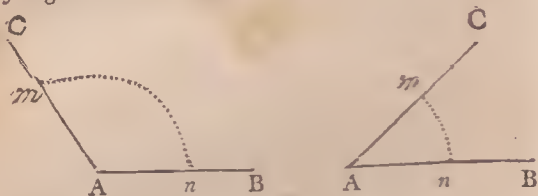
Note. Angles greater than 90 degrees are usually taken at twice.

* The line of chords made use of in the following problems is commonly put upon the plain scale, and is adapted to 90 degrees, or the fourth part of a circle.

For a description of this, and other instruments made use of in Practical Geometry, see Mr. Robertson's *Treatise on such mathematical instruments as are usually put into a portable case*.

PROBLEM XXXV.

Any angle BAC being given, to find the number of degrees it contains.*



1. From the angular point A, with the chord of 60 degrees, describe the arc nm , cutting the legs in the points n and m .

2. Take the distance nm , and apply it to the scale of chords, and it will show the degrees required.

Note: When the distance nm is greater than 90° , it must be taken at twice, as before.

PROBLEM XXXVI.

In a given circle to inscribe a polygon of any proposed number of sides.



* Both this and the last problem may be performed by means of a *protractor*, which is a graduated arc designed for that purpose.

1. Divide 360° by the number of sides, and make an angle AOB, at the centre, whose measure shall be equal to the degrees in the quotient.

2. Join the points A, B, and apply the chord AB to the circumference the given number of times, and it will form the polygon required.

PROBLEM XXXVII.

On a given line AB to form a regular polygon of any proposed number of sides.



1. Divide 360° by the number of sides, and subtract the quotient from 180 degrees.

2. Make the angles ABO and BAO each equal to half the difference last found.

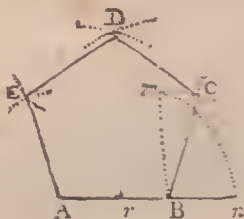
3. From the point of intersection O, with the distance OA or OB, describe a circle.

4. Apply the chord AB to the circumference the proposed number of times, and it will form the polygon required*.

* By this method the circumference of a circle may also be divided into any number of equal parts; for if 360° be divided by the number of parts, and the angle AOB be made equal to the degrees in the quotient, the arc AB will be one of the equal parts required.

PROBLEM XXXVIII*.

Upon a given right line AB to describe a regular pentagon.



1. Produce AB towards n , and at the point B make the perpendicular Bm equal to AB.

2. Bisect AB in r , and from r as a centre, with the radius rm , describe the arc mn , cutting AB in n .

3. From the points A and B, with the radius An, describe arcs cutting each other in D.

4. And from the points A, D and B, D, with the radius AB, describe arcs cutting each other in C and E.

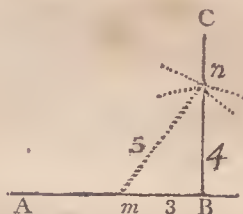
5. Join EC, DC, DE, and EA, and ABCDE will be the pentagon required.

This method differs but little from that of Problem xxviii, and is equally easy and convenient in practice.

* This and the following problem were not given in the first edition of this work, but are now added on account of their elegance and utility. The second is derived from the 47th Prop. B. I. Euclid's Elements, and the first is proposed for a demonstration in the Ladies' Diary for the year 1786.

PROBLEM XXXIX.

To raise a perpendicular from any point B in a given line AB.



1. From any scale of equal parts take a distance equal to 3 divisions, and set it from B to m .
2. And from the points B and m , with the distances 4 and 5, taken from the same scale, describe arcs cutting each other in n .
3. Through the points n , B, draw the line BC, and it will be the perpendicular required.

Explanation of the characters made use of in the following part of the Work.

+	Is the sign of addition.
—	of subtraction.
×	of multiplication.
÷	of division.
✓	of the square root.
∛	of the cube root.
=	of equality.
:::	of proportion.

OF THE
MENSURATION
OF
SUPERFICIES.

THE *area* of any figure is the measure of its surface, or the space contained within the bounds of that surface, without any regard to thickness.

A square whose side is one inch, one foot, or one yard, &c. is called the *measuring unit*, and the area or content of any figure is computed by the number of those squares contained in that figure.

PROBLEM I.

To find the area of a parallelogram; whether it be a square, a rectangle, a rhombus, or a rhomboides.

RULE.

Multiply the length by the perpendicular height, and the product will be the area*.

* **RULE II.** If any two sides of a parallelogram be multiplied together, and the product again by the natural sine of their included angle, the last product will give the area of the triangle. That is, $AB \times BC \times \text{nat. sine of the angle } B = \text{area.}$

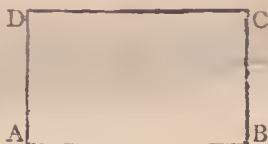
EXAMPLES.

1. Required the area of the square ABCD whose side is 5 feet 9 inches.



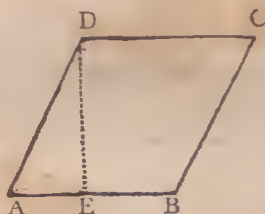
Here $5\text{ ft. } 9\text{ in.} = 5.75$; and $5.75^2 = 5.75 \times 5.75 = 33.0625$, feet = $33\text{ ft. } 0\text{ in. } 9\text{ pa.} = \text{area required.}$

2. Required the area of the rectangle ABCD, whose length AB is 13.75 chains, and breadth BC 9.5 chains.



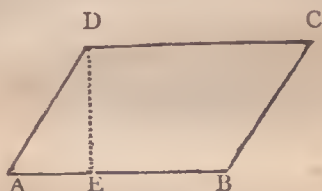
Here $13.75 \times 9.5 = 130.625$; and $\frac{130.625}{10} = 13.0625$ acres = $13\text{ ac. } 0\text{ ro. } 10\text{ po.} = \text{area required.}$

3. Required the area of the rhombus ABCD, whose length AB is 12 ft. 6 in. and its height DE 9 ft. 3 in.



Here 12 *fe.* 6 *in.* = 12.5, and 9 *fe.* 3 *in.* = 9.25 ;
 Whence $12.5 \times 9.25 = 115.625$ *feet* = 115 *fe.* 7 *in.*
 6 *sq. a.* = area required.

4. What is the area of the rhomboides ABCD, whose length AB is 10.52 chains, and height DE 7.63 chains?



Here $10.52 \times 7.63 = 80.2676$; and $\frac{80.2676}{.0} =$
 8.02676 acres = 8 *ac.* 0 *ro.* 4 *po.* = area required.

5. What is the area of a square whose side is 35.25 chains?

ac. ro. po.
 Ans. 124 1 1

6. What is the area of a square whose side is 8 feet 4 inches?

fe. in. sq. a.
 Ans. 69 5 4

7. What is the area of a rectangle whose length is 14 feet 6 inches, and breadth 4 feet 9 inches?

fe. in. sq. a.
 Ans. 68 10 6

3. Required the area of a rhombus, the length of whose side is 12.24 feet, and height 9.16 feet.

fe. in. pa.

Ans. 112 1 5

9. Required the area of a rhomboides whose length is 10.51 chains, and breadth 4.28 chains.

ac. ro. p.

Ans. 4 1 39

10. What is the area of a rhomboides whose length is 7 feet 9 inches, and height 3 feet 6 inches.

fe. in. pa.

Ans. 27 1 6

11. To find the area of a rectangular board, whose length is $12\frac{1}{2}$ feet, and breadth 9 inches.

Ans. $9\frac{3}{8}$ feet.

PROBLEM II.

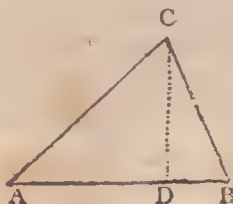
To find the area of a triangle.

RULE.

Multiply the base by the perpendicular height, and half the product will be the area.

EXAMPLES.

1. Required the area of the triangle ABC, whose base AB is 10 feet 9 inches, and height DC 7 feet 3 inches.



Here 10 *fe.* 9 *in.* = 10.75, and 7 *fe.* 3 *in.* = 7.25 ;

Whence $10.75 \times 7.25 = 77.9375$, and $\frac{77.9375}{2} =$

38.96875 *feet* = 38 *fe.* 11 *in.* $7\frac{1}{2}$ *pa.* = area required.

2. What is the area of a triangle whose base is 18 feet 4 inches, and height 11 feet 10 inches ?

fe. in. pa.

Ans. 106 5 8

3. What is the area of a triangle whose base is 16.75 feet, and height 6.24 feet ?

fe. in. pa.

Ans. 52 3 1

4. Required the area of a triangle whose base is 12.25 chains, and height 8.5 chains.

ac. ro. po.

Ans. 5 0 33

5. What is the area of a triangle whose base is 20 feet, and height 10.25 ?

Ans. 102.5 *fe.*

PROBLEM III.

To find the area of a triangle whose three sides only are given.

RULE.

1. From half the sum of the three sides subtract each side severally.

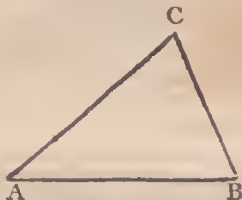
2. Multiply the half sum and the three remainders continually together, and the square root of the product will be the area required*.

* RULE II. Any two sides of a triangle being multiplied together, and the product again by half the natural sine of their included angle, will give the area of the triangle.

That is, $AC \times CB \times \text{nat. sine of the angle } C = \text{twice area.}$

EXAMPLES.

1. Required the area of the triangle ABC whose three sides BC, CA, and AB, are 24, 36, and 48 chains respectively.



Here $\frac{24 + 36 + 48}{2} = \frac{108}{2} = 54 = \frac{1}{2}$ sum of the sides ;

Also $54 - 24 = 30$ first diff. $54 - 36 = 18$ second diff. and $54 - 48 = 6$ third diff.

Whence $\sqrt{54 \times 30 \times 18 \times 6} = \sqrt{174960} = 418.043$
= area required.

2. Required the area of a triangle whose three sides are 13, 14, and 15 feet. *Ans.* 84 square feet.

3. How many acres are there in a triangle whose three sides are 49.00, 50.25 and 25.69 chains?

Ans. 61.498 ac.

4. Required the area of a right-angled triangle, whose hypotenuse is 50, and the other two sides 30 and 40. *Ans.* 600

5. Required the area of an equilateral triangle whose side is 25. *Ans.* 270.625

6. Required the area of an isosceles triangle whose base is 20, and each of its equal sides 15. *Ans.* 111.803

7. Required the area of a triangle whose three sides are 20, 30, and 40 chains. *Ans.* 29 ac. 7 po.

PROBLEM IV.

Any two sides of a right-angled triangle being given to find the third side.

RULE.

1. *When the two legs are given, to find the hypotenuse.*

Add the square of one of the legs to the square of the other, and the square root of the sum will be equal to the hypotenuse.

2. *When the hypotenuse and one of the legs are given, to find the other leg.*

From the square of the hypotenuse take the square of the given leg, and the square root of the remainder will be equal to the other leg.

EXAMPLES.

1. In the right-angled triangle ABC, the base AB is 56, and the perpendicular BC 33: what is the hypotenuse?



Here $56^2 + 33^2 = 3136 + 1089 = 4225$; and $\sqrt{4225} = 65 = \text{hypotenuse AC.}$

2. If the hypotenuse AC be 53, and the base AB 45: what is the perpendicular BC?

Here $53^2 - 45^2 = 2809 - 2025 = 784$; and $\sqrt{784} = 28 = \text{perpendicular BC}$.

3. The base of a right-angled triangle is 77, and the perpendicular 36: what is the hypotenuse?

Ans. 85.

4. The hypotenuse of a right-angled triangle is 109, and the perpendicular 60: what is the base?

Ans. 91.

5. It is required to find the length of a shoar, which strutting 12 feet from the upright of a building, will support a jaumb 20 feet from the ground.

Ans. 23.32380 feet.

6. The height of a precipice, standing close by the side of a river, is 193 feet, and a line of 320 feet will reach from the top of it to the opposite bank; required the breadth of the river.

Ans. 302.9703 feet.

7. A ladder 50 feet long, being placed in a street, reached a window 28 feet from the ground, on one side; and by turning it over, without removing the foot, it reached another window, 36 feet high on the other side: required the breadth of the street.

Ans. 76.1233335 feet.

PROBLEM V.

To find the area of a trapezium.

RULE*.

Multiply the diagonal by the sum of the two per-

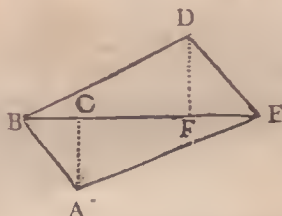
* If the trapezium can be inscribed in a circle; that is, if the sum of two of its opposite angles is equal to two right angles, or 180° , the area may be found thus:

Rule. From half the sum of the four sides subtract each side severally; then multiply the four remainders continually together, and the square root of the product will be the area.

pendiculars falling upon it from the opposite angles, and half the product will be the area.

EXAMPLES.

1. Required the area of the trapezium BAED, whose diagonal BE is 84, the perpendicular AC 21, and DF 28.



Here $\overline{28+21} \times 84 = 49 \times 84 = 4116$; and $\frac{4116}{2} = 2058$ the area required.

2. Required the area of a trapezium whose diagonal is 80.5, and the two perpendiculars 24.5 and 30.1:
Ans. 2197.65.

3. What is the area of a trapezium whose diagonal is 108 feet 6 inches, and the perpendiculars 56 feet 3 inches, and 60 feet 9 inches?
Ans. 6347 fe. 3 in.

PROBLEM VI.

To find the area of a trapezoid, or a quadrangle, two of whose opposite sides are parallel.

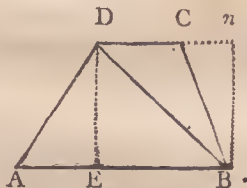
RULE.

Multiply the sum of the parallel sides by the per-

pendicular distance between them, and half the product will be the area.

EXAMPLES.

1. Required the area of the trapezoid ABCD, whose sides AB and DC are 321.51 and 214.24, and perpendicular DE 171.16.



Here $321.51 + 214.24 = 535.75 = \text{sum of the parallel sides AB, DC.}$

Whence 535.75×171.16 (the perp. DE) = 91698.9700;

And $\frac{91698.9700}{2} = 45849.485$ the area required.

The parallel sides of a trapezoid are 12.41 and 8.22 chains, and the perpendicular distance 5.15 chains; required the area.

ac. ro. po.

Ans. 5 1 9

3. Required the area of a trapezoid whose parallel sides are 25 feet 6 inches and 18 feet 9 inches, and the perpendicular distance 10 feet 5 inches.

fe. in. pa.

Ans. 230 5 7

4. Required the area of a trapezoid whose parallel sides are 20.5 and 12.25, and perpendicular distance 10.75.

Ans. 176.03125.

PROBLEM VII.

To find the area of a regular polygon.

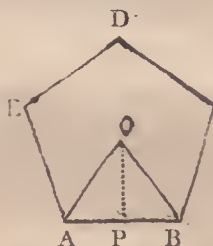
RULE.

i Multiply half the perimeter of the figure by the perpendicular falling from its centre upon one of the sides, and the product will be the area.

Note. The perimeter of any figure is the sum of all its sides.

EXAMPLES.

1. Required the area of the regular pentagon ABCDE, whose side AB, or BC, &c. is 25 feet, and perpendicular OP 17.2 feet.



Here $\frac{25 \times 5}{2} = 62.5 = \text{half perimeter}; \text{ and } 62.5 \times 17.2 = 1075 \text{ square feet} = \text{area required.}$

2. Required the area of a hexagon whose side is 14.6 feet, and perpendicular 12.64 feet.

Ans. 553.632 square feet.

3. Required the area of a heptagon whose side is 19.38, and perpendicular from the centre 20.

Ans. 13566.

4. Required the area of an octagon whose side is 9.941, and perpendicular 12.

Ans. 477.165.

PROBLEM VIII.

To find the area of a regular polygon, when the side only is given.

RULE*.

Multiply the square of the side of the polygon, by the number standing opposite to its name in the following table, and the product will be the area.

No. of sides.	Names.	Multipliers.
3	Trigon or equil. Δ	0.433013—
4	Tetragon or square	1.000000+
5	Pentagon	1.720477+
6	Hexagon	2.598076+
7	Heptagon	3.635912+
8	Octagon	4.828427+
9	Nonagon	6.181824+
10	Decagon	7.694209—
11	Undecagon	9.365640—
12	Duodecagon	11.196152+

* The multipliers in the table are the areas of the polygons to which they belong when the side is unity or 1.

EXAMPLES.

1. Required the area of a pentagon whose side is 15.

$$\begin{array}{r}
 15 \\
 15 \\
 \hline
 75 \\
 15 \\
 \hline
 \end{array}$$

$225 = \text{square of the side.}$

$1.720477 = \text{area, when the side is 1.}$

$$225$$

$$\begin{array}{r}
 8602385 \\
 3440954 \\
 3440954 \\
 \hline
 \hline
 \end{array}$$

$387.107325 = \text{area required.}$

2. The side of a hexagon is 5 feet 4 inches; what is the area? *Ans. 73.9.*
 3. Required the area of an octagon whose side is 16. *Ans. 1236.073.*
 4. Required the area of a decagon whose side is 20.5. *Ans. 3233.4913.*
 5. Required the area of a nonagon whose side is 36. *Ans. 8011.6439.*
 6. Required the area of a duodecagon whose side is 125? *Ans. 174939.875.*

PROBLEM IX.

The diameter of a circle being given, to find the circumference; or, the circumference being given, to find the diameter.

RULE I*.

As 7 is to 22, so is the diameter to the circumference. Or, as 22 is to 7, so is the circumference to the diameter.

* The proportion of the diameter of a circle to its circumference has never yet been exactly attained. Nor can a square, or any other right-lined figure, be found, that shall be equal to a given circle. This is the celebrated problem, called the *squaring of the circle*, which has exercised the abilities of the greatest mathematicians for ages, and been the occasion of so many endless disputes. Several persons of considerable eminence have, at different times, pretended that they had discovered the exact quadrature; but their errors have soon been detected, and it is now generally looked upon as a thing impossible to be done.

But though the relation between the diameter and circumference cannot be accurately expressed in known numbers, it may yet be approximated to any assigned degree of exactness. And in this manner was the problem solved by the great *Archimedes*, about two thousand years ago, who discovered the proportion to be nearly as 7 is to 22, which is the same as our first rule.

This he effected by showing that the perimeter of a circumscribed regular polygon of 192 sides is to the diameter in a less ratio than that of $3\frac{10}{70}$ to 1, and that the perimeter of an inscribed polygon of 96 sides is to the diameter in a greater ratio than that of $3\frac{10}{71}$ to 1, and from thence inferred the ratio above-mentioned; as may be seen in his book *de dimensione circuli*. The same proportion was also discovered by *Philo Gadarensis*, and *Apollonius Pergeus*, at a still earlier period, as we are informed by *Eutocius*, in his observations on a work not come to our hands, called *Ocyteboos*.

The proportion of *Vieta* and *Metius* is that of 113 to 355, which is something more exact than the former, and is the same as the second rule.

This is a very commodious proportion; for being reduced

EXAMPLES.

1. If the diameter AB of a circle be 9, what is the circumference?



Here $7 : 22 :: 9 : 28\frac{2}{7}$; Or $\frac{22 \times 9}{7} = 28\frac{2}{7}$ = circumference required.

into decimals, it agrees with the truth as far as the sixth figure inclusively. It was derived from the pretended quadrature of a M. Van-eick, which first gave rise to the discovery.

But the first who ascertained this ratio to any great degree of exactness was *Van Ceulen*, a *Dutchman*, in his book *de Circulo & Adscriptis*. He found that if the diameter of a circle was 1, the circumference would be 3.141592653589793238 462643383279502884, nearly; which is exactly true to 36 places of decimals, and was effected by means of the continual bisection of an arc of a circle, a method so exceedingly troublesome and laborious, that it must have cost him incredible pains: It is said to have been thought so curious a performance, that the numbers were cut on his tomb-stone, in *St. Peter's Church-yard*, at *Leyden*. This last number has since been confirmed, and extended to double the number of places, by the late ingenious *Mr. Abraham Sharp*, of *Little Horton*, near *Bradford*, in *Yorks ire*.

But since the invention of *Fluxions*, and the *Summation of Infinite Series*, there have been several methods found out for doing the same thing with much more ease and expedition. The late *Mr. John Machin*, *Professor of Astronomy in Gresham College*, has, by these means, given a qua-

2. If the circumference of a circle ADBC be 36 feet, what is the diameter?

$$22 : 7 :: 36$$

7

$$\begin{array}{r} 22 \overline{) 252} (11\frac{10}{22} \text{ feet, the diameter.} \\ 22 \end{array}$$

32

22

10

$$\text{Or } \frac{36 \times 7}{22} = \frac{18 \times 7}{11} = \frac{126}{11} = 11\frac{5}{11}, \text{ as before.}$$

RULE II.

As 113 is to 355 so is the diameter to the circumference. Or, as 355 is to 113 so is the circumference to the diameter.

EXAMPLES.

1. The diameter of a circle is 9 feet; what is the circumference?

$$\text{Here, as } 113 : 355 :: 9 : 28\frac{31}{113}.$$

$$\text{Or } \frac{355 \times 9}{113} = \frac{3195}{113} = 28.27 = \text{circumference.}$$

drature of the circle which is true to 100 places of decimals; and *M. de Lagny*, *M. Euler*, &c. have carried it still farther. All of which proportions are so extremely near the truth, that, except the ratio could be completely obtained, we need not wish for a greater degree of accuracy.

2. If the diameter of a circle be 10 feet, what is the circumference?

$$\text{As } 113 : 355 :: 10$$

$$\begin{array}{r} 113 \overline{) 3550} (31 \frac{4}{113} \text{ feet, the circumference.} \\ \underline{339} \\ 160 \\ \underline{113} \\ 47 \end{array}$$

$$\text{Or } \frac{355 \times 10}{113} = \frac{3550}{113} = 31.41 = \text{circumference.}$$

3. What is the diameter of a circle whose circumference is 50?

$$\text{As } 355 : 113 :: 50$$

$$\begin{array}{r} 355 \overline{) 5650} (15 \text{ the diameter.} \\ \underline{355} \\ 2100 \\ \underline{1775} \\ 325 \end{array}$$

RULE III.

Multiply the diameter by 3.1416, and the product will be the circumference: or,

Divide the circumference by 3.1416, and the quotient will be the diameter.

EXAMPLES.

1. If the diameter of a circle be 17, what is the circumference?

$$\begin{array}{r} 3.1416 \\ 17 \\ \hline 219912 \\ 31416 \\ \hline \end{array}$$

53.4072 = circumference.

2. If the circumference of a circle be 354, what is the diameter?

By Rule I.

As 22 : 7 :: 354 : $\frac{354 \times 7}{22} = 112.639 = \text{circumference.}$

By Rule II.

As 355 : 113 :: 354 : $\frac{354 \times 113}{355} = 112.681 = \text{circumference.}$

By Rule III.

As 3.1416 : 1 :: 354 : $\frac{354}{3.1416} = 112.681 = \text{circumference, agreeing with the former as far as the third place of decimals.}$

3. What is the circumference of a circle whose diameter is 40 feet? *Ans.* 125.6640.

4. If the circumference of a circle be 12, what is the diameter? *Ans.* 3.81972.

5. The earth is a globe, and its circumference is known to be 25000 miles, what is its diameter?

Ans. 7958 nearly.

PROBLEM X.

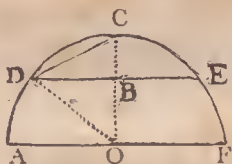
To find the length of any arc of a circle.

RULE I.

From 8 times the chord of half the arc subtract the chord of the whole arc, and $\frac{1}{3}$ of the remainder will be the length of the arc *nearly*.

EXAMPLES.

1. The chord of the whole arc DE is 48, and the versed sine BC of half the arc is 18: what is the length of the arc DCE?



Here $24^2 + 18^2 = 576 + 324 = 900$; and $\sqrt{900} = 30 = DG$. Whence $\frac{30 \times 8 - 48}{3} = \frac{240 - 48}{3} = \frac{192}{3} = 64$ the length of the arc required.

2. The chord of the whole arc is 50.8, and the chord of half the arc is 30.6: required the length of the arc. *Ans.* 64.6.

3. The length of the whole chord is 6, and the length of the chord of half the arc 3.0538: what is the length of the arc? *Ans.* 6.14768.

RULE II.

1. Add the square of half the chord to the square of the versed sine of half the arc, and this sum divided by the versed sine will give the diameter.

2. Take $\frac{4}{3}$ of the versed sine from the diameter, and divide $\frac{2}{3}$ of the versed sine by the remainder.

3. To the quotient, last found, add 1; and this sum multiplied by the chord of the whole arc, will give the length of the arc *nearly*.

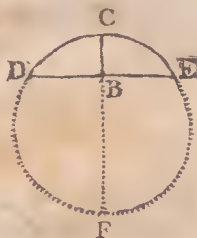
Note. The length of the arc may also be found by multiplying together the number of degrees it contains, the radius, and the number .01745329.

Or, as 180 is to the number of degrees in the arc, so is 3.1416 times the radius, to its length.

Or, as 3 is to the number of degrees in the arc, so is .05236 times the radius, to its length*.

EXAMPLES.

1. If the chord DE be 48, and the versed sine CB 18: what is the length of the arc?



* *Note.* When very great accuracy is required, the following theorem may be used:

Let d denote the diameter of the circle, and v the versed sine of half the arc: then the arc $= 2\sqrt{dv} \times (1 + \frac{v}{6d} + \frac{3v^2}{40d^2} + \frac{5v^3}{112d^3} + \frac{35v^4}{1152d^4} + \frac{63v^5}{2816d^5}, \&c.)$

G

Here $\frac{24^2 + 18^2}{18} = \frac{576 + 324}{18} = \frac{900}{18} = 50 = \text{dia-}$
meter CF.

And $18 \times \frac{2}{3} \div (50 - \frac{41}{50} \times 18) = 12 \div 50 - 14.70 =$
 $12 \div 35.24 = .3405.$

Whence $(1 + .3405) \times 48 = 1.3405 \times 48 = 64.3440$
 = length of the arc required.

2. The chord of the whole arc is 7, and the versed
 sine 2: what is the length of the arc?

Ans. 8.8439.

3. The chord of the whole arc is 40, and the versed
 sine 15: what is the length of the arc?

Ans. 53.62.

PROBLEM XI.

To find the area of a circle.

RULE I.

Multiply half the circumference by half the dia-
 meter, and the product will be the area.

Or take $\frac{1}{4}$ of the product of the whole circumfe-
 rence and diameter.

EXAMPLES.

1. What is the area of a circle whose diameter is
 42, and circumference 131.946?

$$2)131.946$$

$$65.973 = \frac{1}{2} \text{ circumference.}$$

$$21 = \frac{1}{2} \text{ diameter.}$$

$$65973$$

$$131946$$

$$1385.433 = \text{area required.}$$

2. What is the area of a circle whose diameter is 10 feet 6 inches, and circumference 31 feet 6 inches?

fe. in.

$$15 \quad 9 = 15.75 = \frac{1}{2} \text{ circumference.}$$

$$5 \quad 3 = 5.25 = \frac{1}{2} \text{ diameter.}$$

$$7875$$

$$3150$$

$$7375$$

$$82.6875$$

$$12$$

$$8.2500$$

Ans. 82 feet, 8 inches.

3. What is the area of a circle whose diameter is 1, and circumference 3.14159? *Ans. .7854.*

4. What is the area of a circle whose diameter is 7, and circumference 22? *Ans. $38\frac{1}{2}$.*

RULE II*.

Multiply the square of the diameter by .7854, and the product will be the area.

Or, multiply the square of the circumference by .07958, the product will be the area.

EXAMPLES.

1. What is the area of a circle whose diameter is 5?

$$.7854$$

25 = square of the diameter.

$$39270$$

$$15708$$

19.6350 = the answer.

2. What is the area of a circle whose diameter is 7? Ans. 38.4846.

3. What is the area of a circle whose diameter is 4.5? Ans. 15.90435.

* The following table will also show most of the useful problems, relating to the circle and its equal or inscribed square.

1. diameter $\times .8862$ = side of an equal square.
2. circumf. $\times .2821$ = side of an equal square.
3. diameter $\times .7071$ = side of the inscribed square.
4. circumf. $\times .2251$ = side of the inscribed square.
5. area $\times .6366$ = side of the inscribed square.
6. side of a square $\times 1.4142$ = diameter of its circums. circle.
7. side of a square $\times 4.443$ = circumf. of its circums. circle.
8. side of a square $\times 1.128$ = diameter of an equal circle.
9. side of a square $\times 3.545$ = circumf. of an equal circle.

4. How many square yards are there in a circle whose diameter is $3\frac{1}{2}$ feet?

Ans. 1.069016.

5. How many square feet are there in a circle, whose circumference is 10.9956 yards?

Ans. 86.19266.

6. How many square perches are there in a circle the circumference of which is 7 miles?

Ans. 399300.608.

PROBLEM XII.

To find the area of a sector, or that part of a circle which is bounded by any two radii and their included arc.

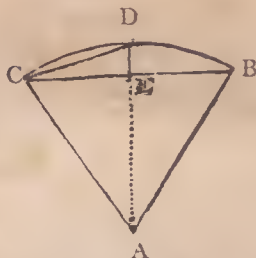
RULE.

Multiply the radius, or half the diameter, by half the length of the arc of the sector, and the product will be the area.

Note. The length of the arc may be found by either of the last problems.

EXAMPLE

1. The radius AB is 40, the chord BC of the whole arc 50, and the chord CD of half the arc 27; required the area of the sector.



Here $\frac{27 \times 8 - 50}{3} = \frac{216 - 50}{3} = \frac{166}{3} = 55.\dot{3} =$
length of the arc CDB.

And $\frac{55.\dot{3}}{2} \times 40 = 27.\dot{6} \times 40 = 1109.\dot{6} =$ area of the
sector required.

2. Required the area of a sector of a circle, whose radius is 25, and the length of the arc 21.5.

Ans. 268.75.

3. What is the area of a sector, the chord of the whole arc being 20, and the chord of half the arc 11?

Ans. 149.507.

4. Find the area of the sector whose radius is 9, and the chord of whose arc is 6.

Ans. 27.5267835.

5. The chord of the whole arc is 40, and the versed sine of half the arc 15: what is the area of the circle?

Ans. 558.544.

6. Required the area of the sector; the diameter of the circle being 100, and the chord of half the arc 30.

Ans. 1523.59.

RULE II.

As 360 is to the degrees in the arc of a sector, so is the area of the whole circle, whose radius is equal

to that of the sector, to the area of the sector required.

Note. For a semicircle, a quadrant, &c. take one half, one quarter, &c. of the whole area.

EXAMPLES.

1. The radius of a sector of a circle is 20, and the degrees in its arc 22; what is the area of the sector?

$$\begin{array}{r} .7854 \\ 1600 = \text{square of the diameter.} \end{array}$$

$$\begin{array}{r} 4712400 \\ 7854 \end{array}$$

$$360 : 22 :: 1256.6400 = \text{area of the whole circle.}$$

$$\begin{array}{r} 22 \\ \hline 251328 \\ 251328 \end{array}$$

$$36.0)2764.608(76.794 = \text{area of the sector.}$$

$$\begin{array}{r} 252 \\ \hline 244 \\ 216 \\ \hline 286 \\ \text{\&c.} \end{array}$$

2. Required the area of a sector whose radius is 25, and the length of its arc 147 degrees 29 minutes.

Ans. 804 3986.

3. Required the area of a semicircle whose radius is 13.

Ans. 265.4652.

4. Required the area of a quadrant whose radius is 21. *Ans.* 346.3614.

PROBLEM XIII.

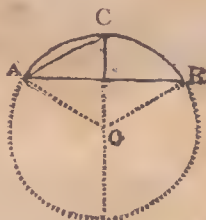
To find the area of a segment of a circle.

RULE I.

1. Find the area of the sector, having the same arc with the segment, by one of the last problems.
2. Find the area of the triangle formed by the chord of the segment, and the radii of the sector.
3. Then the sum, or difference, of these areas, according as the segment is greater or less than a semicircle, will be the area required.

EXAMPLES.

1. The radius OB is 10, the chord AB is 18, and the chord AC 10; what is the area of the segment ABC?



Here $\frac{10 \times 8 - 18}{3} = \frac{80 - 18}{3} = \frac{62}{3} = 20.6 = \text{arc}$
ACB.

And $\frac{20.6}{2} \times 10 = 10.3 \times 10 = 103.3 = \text{area of the sector OACB.}$

Also $\frac{18+10+10}{2} = \frac{38}{2} = 19 = \frac{1}{2} \text{ sum of the sides AO, OB, BA.}$

Whence $19-10=9=1\text{st and } 2\text{d diff. and } 19-18=1=3\text{d diff.}$

And $\sqrt{19 \times 9 \times 9 \times 1} = \sqrt{1539} = 39.23 = \text{area of the } \triangle BOA.$

And $103.3-39.23=64.10=\text{area of the segment required.}$

It being in this case less than a semicircle.

2. Required the area of a segment whose height is 2, and chord 20. *Ans.* 26.878.

3. Required the area of a segment of a circle whose radius is 24, the chord of the whole arc 20, and the chord of half the arc 10.2. *Ans.* 28.07.

4. Required the area of a segment of a circle whose chord is 12, and the radius of the circle 10. *Ans.* 16 3274.

5. Required the area of a segment of a circle whose chord is 16, and the diameter of the circle 20. *Ans.* 44.7292.

6. What is the area of a segment whose arc is a quadrant, the diameter being 18 feet? *Ans.* 23.1174.

7. What is the area of a segment of a circle whose arc contains 280 degrees, the diameter being 50? *Ans.* 1834.9191.

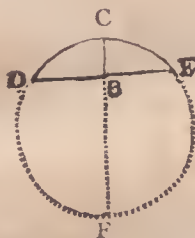
RULE II.

1. Add the square of half the chord of the segment to the square of its height, and multiply the square root of the sum by 4.

2. To $\frac{1}{3}$ of the number last found, add the whole chord of the segment, and this sum multiplied by $\frac{2}{5}$ of the height will give the area required.

EXAMPLES.

1. The chord DE of the segment DEC is 40, and its height, or versed sine, BC is 10 : what is the area of the segment?



$$\text{Here } 4\sqrt{20^2 + 10^2} = 4\sqrt{400 + 100} = 4\sqrt{500} = 4 \times 22.36 = 89.44.$$

$$\text{And } \left(\frac{89.44}{3} + 40\right) \times \frac{2 \times 10}{5} = 29.81 + 40 \times 4 = 69.81 \times 4 = 279.24 = \text{area required.}$$

2. The chord is 20, and the height, or versed sine 2; required the area of the segment.

Ans. 26,85908.

3. The length of the chord is 48, and the height of the segment 18 : what is the area?

Ans. 633.6.

RULE III.

1. Divide the height, or versed sine, by the diameter, and find the quotient in the table of versed sines.

2. Multiply the number on the right hand of the versed sine, by the square of the diameter, and the product will be the area.

EXAMPLES.

1. If the chord of a circular segment be 40, its versed sine 10, and the diameter of the circle 50: what is the area?

$$5.0)1.0$$

.2=tabular versed sine.

.111823=tabular segment.

2500=square of 50.

$$55911500$$

$$223646$$

$$279.557500=\text{area required.}$$

2. The chord of the segment is 20, the versed sine 5, and the diameter 25: what is the area?

Ans. 69.889375.

3. The diameter of a circle is 40, and the versed sine 10: what is the area?

Ans. 245.6736.

4. If the diameter be 52, and the versed sine 2: what is the area?

Ans. 26.815.

PROBLEM XIV.

To find the area of a circular zone, or the space included between any two parallel chords and their intercepted arcs.

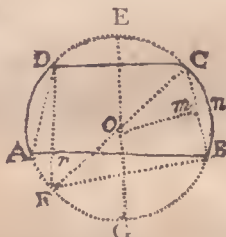
RULE.

1. Find the area of that part of the zone which forms a trapezoid by problem 6, and the area of the segment $mnCB$ by problem 18, rule 3.

2. Add the area of the trapezoid to twice the area of the segment, and it will give the area of the zone required.

EXAMPLES.

1. The greater chord AB is 20, the less DC 15, and their distance Dr $17\frac{1}{2}$: required the area of the zone $ABCD$.



$$\text{Here } \sqrt{Dr^2 + rA^2} = \sqrt{17.5^2 + 2.5^2} =$$

$$\sqrt{306.25 + 6.25} = \sqrt{312.5} = 17.6776 = AD \text{ or } CB.$$

$$\text{And } Dr (17\frac{1}{2}) : rA (2\frac{1}{2}) :: Br (17\frac{1}{2}) : 2\frac{1}{2} = rF; \text{ or } 2\frac{1}{2} + 17\frac{1}{2} = 20 = DF.$$

$$\text{Also } \sqrt{DF^2 + DC^2} = \sqrt{20^2 + 15^2} = \sqrt{400 + 225} = \sqrt{625} = 25 = CF; \text{ or } \frac{25}{2} = 12\frac{1}{2} = OC \text{ or } On.$$

$$\text{And } rD (17\frac{1}{2}) : AD (17.6776) :: rB (17\frac{1}{2}) : 17.6776 = FB; \text{ or } \frac{17.6776}{2} = 8.8388 = Om; \text{ and } 12.5 - 8.8388 = 3.6612 = mn = \text{height of the segment.}$$

Therefore $\frac{3.6612}{25} = .146448 = \text{tab. versed sine; and}$

$.071033 = \text{tab. segment.}$

Whence $.071033 \times 25^2 = .071033 \times 625 = 44.395625$
 $= \text{area of the segment BnC.B.}$

Also $\frac{15+20}{2} \times 17\frac{1}{2} = \frac{35}{2} \times \frac{35}{2} = \frac{1225}{4} = 306.25 =$
 $\text{area of the trapezoid ABCD.}$

And $306.25 + 44.395625 \times 2 = 306.25 + 88.79125$
 $= 394.04125 = \text{area of the zone required.}$

2. The greater side is 96, the lesser 60, and the breadth 26: what is the area of the zone?

Ans. 2136.7712.

3. One end of a circular zone is 48, the other end is 30, and the breadth is 13: what is the area of the zone*?

Ans. 534.4249.

PROBLEM XV.

To find the area of a circular ring, or the space included between the circumferences of two concentric circles.

RULE†.

The difference between the areas of the two circles will be the area of the ring.

* In any of these questions, when the radius or diameter of the circle is given, the operation is much more easily performed.

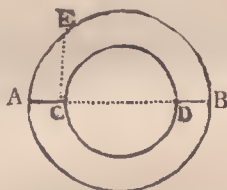
† *Rule 2.* Multiply half the sum of the circumferences by half the difference of the diameters, and the product will be the area.

This rule will also serve for any part of the ring, using half the sum of the intercepted arcs for half the sum of the circumferences.

Or, Multiply the sum of the diameters by their difference, and this product again by .7854, and it will give the area required.

EXAMPLES.

1. The diameters AB and CD are 20 and 15: required the area of the circular ring, or the space included between the circumferences of those circles.



Here $\overline{AB + CD} \times \overline{AB - CD} = 35 \times 5 = 175$; and $175 \times .7854 = 137.4450 = \text{area of the ring required.}$

2. The diameters of two concentric circles are 16 and 10: what is the area of the ring formed by those circles? *Ans.* 122.5224.

3. The two diameters are 21.75 and 9.5: required the area of the circular ring. *Ans.* 300.6609.

5. Required the area of the ring, the diameters of whose bounding circles are 6 and 4. *Ans.* 15.708.

PROBLEM XVI.

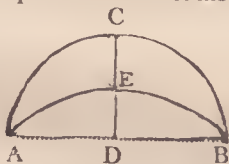
To find the areas of lunes, or the spaces included between the intersecting arcs of two eccentric circles.

RULE*.

Find the areas of the two segments from which the lune is formed, and their difference will be the area required.

EXAMPLES.

The length of the chord AB is 40, the height DC 10, and DE 4: required the area of the lune ACBEA.



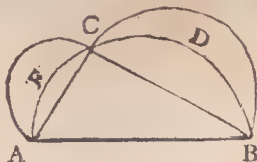
Here $4\sqrt{AD^2 + DC^2} = 4\sqrt{400 + 100} = 22.36 \times 4 = 89.44$.

And $(\frac{89.44}{3} + 40) \times \frac{2 \times 10}{5} = 29.81 + 40 \times 4 = 279.24$
 $= \text{area of the segment ABCA}.$

* Whoever wishes to be acquainted with the properties of lunes, and the various theorems arising from them, may consult *Mr. Whiston's Commentary on Tacquet's Euclid*, where they will find this subject very ingeniously managed.

The following property is one of the most curious.

If ABC be a right-angled triangle, and semicircles be described on the three sides as diameters, then will the said triangle be equal to the two lunes D and F taken together.



Also $4\sqrt{AD^2 + DE^2} = 4\sqrt{400 + 16} = 20.396 \times 4 = 81.584.$

And $(\frac{81.584}{3} + 40) \times \frac{2 \times 4}{5} = \frac{27.194 + 40}{1} \times 1.6 = 107.5104 = \text{area of the segment ABEA}.$

Whence $279.24 - 107.5104 = 171.7296 = \text{area of the lune required}^*.$

2. The chord is 20, and the heights of the segments 10 and 2: required the area of the lune.

Ans. 128.555.

3. The length of the chord is 48, and the heights of the segments 18 and 7: what is the area of the lune?

Ans. 405.8676.

PROBLEM XVII.

To find the area of an irregular polygon, or a figure of any number of sides.

RULE.

1. Divide the figure into triangles and trapeziums, and find the area of each separately.

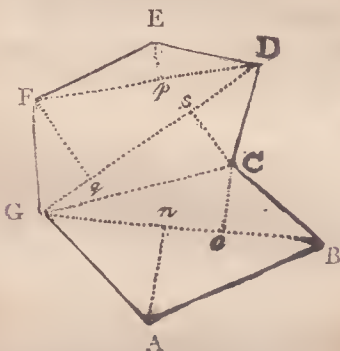
2. Add these areas together, and the sum will be equal to the area of the whole polygon.

* *Note.* The areas of the segments in this and the following examples may be more accurately found by Rule 3, page 82.

EXAMPLES.

1. Required the area of the irregular figure ABC DEFGA, the following lines being given.

$$\begin{array}{lll} \text{GB} = 30.5, & \text{An} = 11.2, & \text{Co} = 6 \\ \text{GD} = 29 & \text{Fq} = 11 & \text{Cs} = 6.6 \\ \text{FD} = 24.1 & \text{Eph} = 4 & \text{---} \end{array}$$



$$\begin{aligned} \text{Here } \frac{\text{An} + \text{Co}}{2} \times \text{GB} &= \frac{11.2 + 6}{2} \times 30.5 = 8.6 \times 30.5 \\ &= 262.3 = \text{area of the trapezium ABCG.} \\ \text{And } \frac{\text{Fq} + \text{Cs}}{2} \times \text{GD} &= \frac{11 + 6.6}{2} \times 29 = 8.8 \times 29 = \\ 255.2 &= \text{area of the trapezium GCDF.} \\ \text{Also } \frac{\text{FD} \times \text{Eph}}{2} &= \frac{24.1 \times 4}{2} = \frac{96.4}{2} = 48.2 = \text{area of} \\ \text{the triangle FDE.} \end{aligned}$$

Whence $262.3 + 255.2 + 48.2 = 565.7 = \text{area of the whole figure required.}$

2. * In a pentanglar field, beginning with the

* Note. As this figure consists of three triangles, all of whose sides are given, by calculating their areas accord

south side, and measuring round towards the east, the first, or south side, is 2735 links, the second 3115, the third 2370, the fourth 2925, and the fifth 2220; also the diagonal from the first angle to the third is 3800 links, and that from the third to the fifth 4010: required the area of the field.

Ans. 117 ac. 2 ro. 28 po.

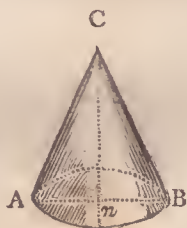
ing to problem 3, the sum will be the area of the whole figure accurately, without drawing perpendiculars from the angles to the diagonals.

The same thing may also be done in most other cases of this kind.

OF THE
CONIC SECTIONS.

DEFINITIONS.

1. **T**HE *conic sections* are such plane figures as are formed by the cutting of a cone.
2. * A *cone* is a solid described by the revolution of a right-angled triangle about one of its legs, which remains fixed.



* This is Euclid's definition of a cone, and is that which is generally best understood by a learner; but the following one is more general.

Conceive the right line *CB* to move upon the fixed point *C* as a centre, and so as continually to touch the circumference of the circle *AB*, placed in any position, except in that of a plane which passes through the said point; and then that part of the line which is intercepted between the fixed point and the periphery of the circle will generate the convex superficies of a cone.

3. The *axis of a cone* is the right line about which the triangle revolves.

4. The *base of a cone* is the circle which is described by the revolving leg of the triangle.

5. If a cone be cut through the vertex, by a plane perpendicular to that of the base, the section will be a *triangle*.



6. If a cone be cut into two parts, by a plane parallel to the base, the section will be a *circle*.



7. If a cone be cut by a plane which passes through its two slant sides in an oblique direction, the section will be an *ellipsis*.



8. If two lines be drawn through the centre of any

ellipses, perpendicular to each other, and terminated both ways by the circumference, they are called the *transverse* and *conjugate diameters*, or *axes*.

The longest line is the transverse axis, and the shortest the conjugate.



9. An *ordinate* is a right line drawn from any point of the curve, perpendicular to either of the diameters.



10. An *abscissa* is that part of the diameter which is contained between the vertex and the ordinate.

11. If a cone be cut by a plane, which is parallel to either of its slant sides, the section will be a *parabola*.



12. The *axis* of a *parabola* is a right line drawn from the vertex, so as to divide the figure into two equal parts.



13. *The ordinate* is a right line drawn from any point in the curve perpendicular to the axis.

14. *The abscissa* is that part of the axis which is contained between the vertex and the ordinate.

15. *If a cone be cut into two parts. by a plane, which being continued, would meet the opposite cone, the section is called an *hyperbola*.



16. *The transverse diameter* or axis of an hyperbola is that part of the axis intercepted between the two opposite cones.

17. *The conjugate diameter* is a line drawn through the centre perpendicular to the transverse.

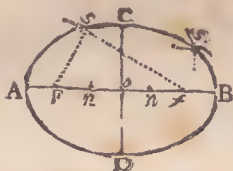
18. *An ordinate* is a line drawn from any point in the curve perpendicular to the axis; and the *abscissa* is the distance intercepted between that ordinate and the vertex.

* The two opposite cones, in this definition, are supposed to be generated together, by the revolution of the same line.

All the figures which can possibly be formed by the cutting of a cone, are mentioned in these definitions, and are the five following ones; viz. a *triangle*, a *circle*, an *ellipsis*, a *parabola*, and an *hyperbola*; but the three last only are usually called the *conic sections*.

PROBLEM I.

To describe an ellipsis, the transverse and conjugate diameters being given.



*Construction**. 1. Draw the transverse and conjugate diameters, AB, CD, bisecting each other perpendicularly in the centre o .

2. With the radius Ao , and centre C , describe an arc, cutting AB in Ff ; and these two points will be the foci of the ellipse.

3. Take any number of points nn , &c. in the transverse diameter AB, and with the radii An , nB , and centres Ff , describe arcs intersecting each other in s , s , &c.

* It is a known property of the ellipse, that the sum of two lines drawn from the foci, to meet in any point in the curve, is equal to the transverse diameter, and from this the truth of the construction is evident.

From the same principle is also derived the following method of describing an ellipse, by means of a string and two pins.

Having found the foci F, f , as before, take a thread of the length of the transverse diameter, and fasten its ends with two pins in the points F, f ; then stretch the thread Fsf to its greatest extent, and it will reach to the point s in the curve; and by moving a pencil round within the thread, keeping it always stretched, it will trace out the curve required.

4. Through the points s, s , &c. draw the curve $AsCBD$, and it will be the circumference of the ellipse required.

PROBLEM II.

In an ellipsis, any three of the four following terms being given, viz. the transverse and conjugate diameters, an ordinate and its abscissa, to find the fourth.

CASE I.

When the transverse, conjugate, and abscissa are known, to find the ordinate.

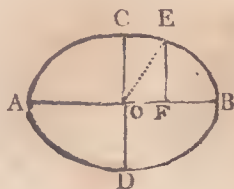
RULE.

As the transverse diameter is to the conjugate,
So is the square root of the rectangle of the two
abscissas,

To the ordinate which divides them.

EXAMPLES.

1. In the ellipsis $ADBC$, the transverse diameter AB is 120, the conjugate diameter CD is 40, and the abscissa BF 24: what is the length of the ordinate EF ?



Here $120 (AB) : 40 (CD) :: \sqrt{96 \times 24} (AF \times FB) :$
 $\frac{40}{120} \sqrt{96 \times 24} = \frac{1}{3} \sqrt{2304} = \frac{1}{3} \times 48 = 16 = EF$ the
 ordinate required.

2. If the transverse diameter be 35, the conjugate 25, and the abscissa 28 : what is the ordinate?

Ans. 10.

CASE II.

When the transverse, conjugate, and ordinate are known, to find the abscissa.

RULE*.

As the conjugate diameter is to the transverse,
 So is the square root of the difference of the
 squares of the ordinate and semi-conjugate,

To the distance between the ordinate and centre.

And this distance being added to and subtracted
 from the semi-transverse, will give the two abscissas
 required.

EXAMPLES.

1. The transverse diameter AB is 120, the conjugate diameter CD is 40, and the ordinate FE is 16; what is the abscissa FB?

* This rule in algebraic terms is as follows: The greater abscissa $x = t + \frac{t}{c} \sqrt{\frac{1}{4}c^2 - y^2}$, or the less $x = t - \frac{t}{c} \sqrt{\frac{1}{4}c^2 - y^2}$.

Here $40 (CD) : 120 (AB) :: \sqrt{20^2 - 16^2}$

$$(\sqrt{OC^2 - FE^2}) : \frac{120}{40} \sqrt{20^2 - 16^2} = 3 \sqrt{400 - 256} \\ = 3 \sqrt{144} = 3 \times 12 = 36 = OF, \text{ the distance from the centre.}$$

Whence $60 (OB) - 36 (OF) = 24 = BF$ }
 And $60 (OA) + 36 (OF) = 96 = AF$ } = two
 abscissas required.

2. What are the two abscissas to the ordinate 10, the diameters being 35 and 25?

Ans. 7 and 28.

CASE III.

When the conjugate, ordinate, and abscissa are known, to find the transverse.

RULE.

1. To, or from, the semi-conjugate, according as the greater or less abscissa is used, add or subtract the square root of the difference of the squares of the ordinate and semi-conjugate.

Then, as this sum or difference is to the abscissa, So is the conjugate to the transverse.

EXAMPLES.

1. The conjugate diameter CD is 40, the ordinate EF is 16, and the abscissa FB 24 : required the transverse AB.

$$\text{Here } 20 - \sqrt{20^2 - 16^2} (\sqrt{OC^2 - EF^2}) = 20 - 12 \\ = 8.$$

And $8 : 24 :: 40 : 120$, the transverse diameter required.

2. If an ordinate and its lesser abscissa be 10 and 7, and the conjugate 25 : what is the transverse?

Ans. 35.

CASE IV.

The transverse, ordinate, and abscissa being given, to find the conjugate.

RULE.

As the square root of the product of the two abscissas is to the ordinate,

So is the transverse diameter to the conjugate.

EXAMPLES.

1. The transverse AB is 120, the ordinate EF 16, and the abscissa FB 24 : required the conjugate.

Here $\sqrt{24 \times 96} (\sqrt{BF \times AF}) : 16 (EF) :: 120 (AB) : 16 \times 120 \div \sqrt{24 \times 96} = 16 \times 120 \div \sqrt{2304}$
 $= 16 \times 120 \div 48 = \frac{15 \times 120}{48} = \frac{120}{3} = 40$ *the conjugate diameter required.*

2. The transverse diameter is 35, the ordinate is 10, and its abscissa 7 : what is the conjugate?

Ans. 25.

PROBLEM III.

To find the circumference of an ellipse, the transverse and conjugate diameters being known.

RULE*.

Multiply the square root of half the sum of the squares of the two diameters by 3.1416, and the product will be the circumference *nearly*.

EXAMPLES.

1. The transverse diameter is 24, and the conjugate 20 : required the circumference of the ellipse.

$$\text{Here } \sqrt{\frac{AB^2 + CD^2}{2}} = \sqrt{\frac{24^2 + 20^2}{2}} = \sqrt{\frac{576 + 400}{2}} \\ = \sqrt{288 + 200} = \sqrt{488} = 22.09.$$

And $22.09 \times 3.1416 = 69.397944$, the circumference required.

2. The axes are 24 and 18 : what is the circumference? *Ans.* 66.6433.

* Rule 2. Multiply $\frac{1}{2}$ the sum of the two diameters by 3.1416, and the product will give the circumference *exact enough to answer most practical purposes*.

Rule 3. Find the circumference both from the last rule and that given above, and $\frac{1}{2}$ the sum of the results will give the circumference *extremely near*.

Note. If a = semi-transverse BO, c = semi-conjugate CO, and x = distance OF, of the ordinate EF from the centre, then will the arc CE be $= x \times (1 + \frac{c^2}{6a^4} x^2 + \frac{4a^2c^2 - c^4}{40a^8} x^4 + \frac{8a^4c^2 - 4a^2c^4 + c^6}{112a^{12}} x^6 \&c.)$

The following may serve as a practical rule for finding the length of the arc CE.

Find the length of a circular arc intercepted by OE and OC, and whose radius is $\frac{1}{2}$ the sum of OE and OC, and it will be the elliptic arc CE *nearly*.

PROBLEM IV.

To find the area of an ellipse, the transverse and conjugate diameters being given.

RULE.

Multiply the transverse diameter by the conjugate, and the product again by .7854, and the result will be the area.

Or, multiply .7854 by one axe, and the product by the other*.

EXAMPLES.

1. What is the area of an ellipse whose transverse diameter is 24, and the conjugate 18?

Here $24 \times 18 \times .7854 = 339.2928 = \text{area required.}$

Or,

.7854

24 = 1867800000.

31416

15708

18.8496

18 = conjugate.

1507968

188496

339.2928 = area before.

* The ellipse is equal to a circle, whose diameter is a mean proportional between the axes of the ellipse.

2. If the axes of an ellipse be 35 and 25: what is the area? *Ans.* 687.225.

3. Required the area of an ellipsis whose two axes are 70 and 50. *Ans.* 2748.9.

PROBLEM V.

To find the area of an elliptic segment, whose base is parallel to either of the axes of the ellipse.

RULE.

1. Divide the height of the segment by that axe of the ellipse of which it is a part, and find, in the table, a circular segment whose versed sine is equal to the quotient.

2. Multiply the segment thus found, and the two axes of the ellipse continually together, and the product will give the area required*.

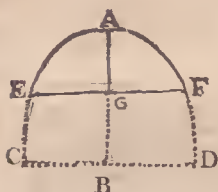
EXAMPLES.

1. Required the area of the elliptic segment EAF, whose height AG is 10, and the axes of the ellipse 2AB and CD, 35 and 25 respectively.

* The area of an elliptic segment may also be found by the following rule.

Find the corresponding segment of the circle described upon the same axe to which the base of the segment is perpendicular.

Then as this axe is to the other axe, so is the circular segment to the elliptic segment.



Here $\frac{10.0000}{35} = \frac{2.0000}{7} = .2857 = \text{tabular versed sine.}$

And the tabular segment belonging to this is .185153.

Whence $.185153 \times 35 (2AB) \times 25 (CD) = 6.480355 \times 25 = 162.0210 = \text{area of the segment required.}$

2. What is the area of an elliptic segment cut off by a double ordinate parallel to the conjugate axe, at the distance of 36 from the centre, the axes being 120 and 40?

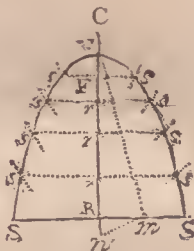
Ans. 536.7504.

3. What is the area of a segment, cut off by an ordinate parallel to the transverse diameter, whose height is 5, the axes being 35 and 25?

Ans. 97.845725.

PROBLEM VI.

To describe a parabola, any ordinate to the axe and its abscissa being given.



** Construction.* 1. Bisect the given ordinate RS in m ; join Vm , and draw mn perpendicular to it, meeting the axis in n .

2. Make VC and VF each equal to Rn , and F will be the focus of the curve.

3. Take any number of points $r, r, \&c.$ in the axis, through which draw the double ordinates $SrS, SrS, \&c.$ of an indefinite length.

4. With the radii $CF, Cr, \&c.$ and centre F , describe arcs cutting the corresponding ordinates in the points $s, s, \&c.$ and the curve SVS drawn through all the points of intersection will be the parabola required.

Note. The line sFs passing through the focus F is called the parameter.

• Besides the methods above, for finding the focus, it may be found arithmetically as follows :

Divide the square of the ordinate by 4 times the abscissa, and the quotient will be the focal distance VF .

Several other methods of doing this, as well as of describing the curve itself, may also be found in Emerson's Conic Sections, and other performances.

PROBLEM VII.

In a parabola, any three of the four following terms being given, viz. any two ordinates and their two abscissas, to find the fourth.

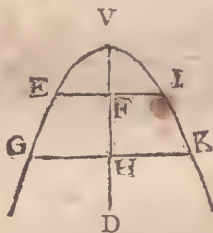
RULE.

As any abscissa is to the square of its ordinate, so is any other abscissa to the square of its ordinate.

Or as the square root of any abscissa is to its ordinate, so is the square root of any other abscissa to its ordinate.

EXAMPLES.

1. The abscissa VF is 9, and its ordinate EF 6; required the ordinate GH, whose abscissa VH is 16.



Here $\sqrt{9}$ (\sqrt{VF}) : 6 (EF) :: $\sqrt{16}$ (\sqrt{VH}) :

$$\frac{6 \times \sqrt{16}}{\sqrt{9}} = \frac{6 \times 4}{3} = \frac{24}{3} = 8 = \text{ordinate GH.}$$

Or,

$$9 (VF) : 36 (EF^2) :: 16 (VH) : \frac{16 \times 36}{9} = 16 \times 4 = 64 = GH^2, \text{ or } 8 = GH, \text{ as before.}$$

2. The two abscissas are 9 and 16, and their corresponding ordinates 6 and 8; from any three of these to find the fourth.

PROBLEM VIII.

To find the length of any arc of a parabola, cut off by a double ordinate.

RULE*.

To the square of the ordinate add $\frac{4}{3}$ of the square of the abscissa, and twice the square root of the sum will be the length of the curve required.

EXAMPLES.

1. The abscissa VH is 2, and its ordinate GH 6: what is the length of the arc GVK?

$$\text{Here } 2^2 (VH^2) \times \frac{4}{3} + 36 (GH^2) = \frac{4 \times 4}{3} + 36 = \frac{16}{3} + 36 = 5.333, \&c. + 36 = 41.333333.$$

$$\text{And } 41.333333(6.429$$

36

2

$$\begin{array}{r} 124 \overline{) 533} \\ 496 \end{array}$$

$$12.858 = \text{length of the arc.}$$

$$\begin{array}{r} 1282 \overline{) 3733} \\ 2564 \end{array}$$

$$\begin{array}{r} 12849 \overline{) 116933} \\ 115941 \end{array}$$

$$11292$$

* *Note.* This rule must be used only when the abscissa does not exceed half the ordinate. The length of the curve in other cases must be found by means of hyperbolic logarithms, as is shown by writers on fluxions.

PROBLEM IX.

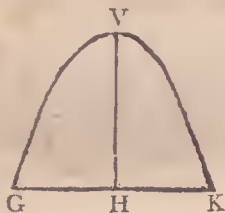
To find the area of a parabola, its base and height being given.

RULE.

Multiply the base by the height, and $\frac{2}{3}$ of the product will be the area required.

EXAMPLES.

1. What is the area of the parabola GVK, whose height VH is 12, and the base or double ordinate GK 16?



Here $16 \text{ (GK)} \times 12 \text{ (VH)} \times \frac{2}{3} = \frac{16 \times 12 \times 2}{3} = 16 \times 4 \times 2 = 128 = \text{area required.}$

2. The abscissa is 12, and the double ordinate or base 38: what is the area? *Ans.* 304.

3. What is the area of a parabola whose abscissa is 10, and ordinate 8? *Ans.* $106\frac{2}{3}$.

PROBLEM X.

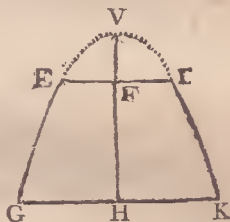
To find the area of the frustum of a parabola.

RULE*.

Divide the difference of the cubes of the two ends of the frustum by the difference of their squares, and this quotient multiplied by $\frac{2}{3}$ of the altitude, will give the area required.

EXAMPLES.

1. In the parabolic frustum GEIK, the two parallel ends EI and GK are 6 and 10, and the altitude, or part of the abscissa FH, is 3: what is the area?



* *Note.* Any parabolic frustum is equal to a parabola of the same altitude, whose base is equal to one end of the frustum, increased by a third proportional to the sum of the two ends and the other end.

$$\text{Here } \frac{10^3 - 6^3}{10^2 - 6^2} (GK^3 - EI^3) \div \frac{10^2 - 6^2}{10^2 - 6^2} (GK^2 - EI^2) \\ = \frac{10^3 - 6^3}{10^2 - 6^2} = \frac{1000 - 216}{100 - 36} = \frac{784}{64} = \frac{98}{8} = \frac{49}{4} = 12.25.$$

And $12.25 \times \frac{2 \times 3}{.3} = 12.25 \times 2 = 24.5 = \text{area required.}$

2. The greater end of the frustum is 24, the lesser end is 20, and their distance $5\frac{1}{2}$: what is the area? *Ans.* 121.3.

3. Required the area of the parabolic frustum, the greater end of which is 10, the less 6, and the height 4. *Ans.* $32\frac{2}{3}$.

PROBLEM XI.

To construct an hyperbola, the transverse and conjugate diameters being given.



Construction. 1. Make AB the transverse diameter, and CD perpendicular to it, the conjugate.

2. Bisect AB in O, and from O with the radius OC, or OD, describe the circle DfCF; cutting AB produced in F and f, which points will be the two foci.

3. In AB produced take any number of points, n, n , &c. and from F and f , as centres, with the distances Bn, An , as radii, describe arcs cutting each other in s, s , &c.

4. Through the several points s, s , &c. draw the curve sBs , and it will be the hyperbola required.

5. If straight lines be drawn from the point O, through the extremities CD of the conjugate axis, they will be the asymptotes of the hyperbola, whose property it is to approach continually to the curve, and yet never to meet it.

PROBLEM XII.

In an hyperbola, any three of the four following terms being given, viz. the transverse and conjugate diameters, an ordinate and its abscissa, to find the fourth.

CASE I.

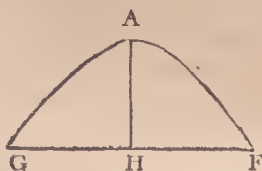
The transverse and conjugate diameters, and the two abscissas being known, to find the ordinate.

RULE.

As the transverse diameter,
Is to the conjugate;
So is the square root of the product of the two
abscissas,
To the ordinate required.

EXAMPLES.

1. In the hyperbola GAF, the transverse diameter is 120, the conjugate 72, and the abscissa AH is 40: required the ordinate FH.



$$\begin{aligned} 120 \text{ (trans.)} : 72 \text{ (conj.)} &:: \sqrt{(160 \times 40)} : \\ \frac{72 \times \sqrt{(160 \times 40)}}{120} &= \frac{6 \times \sqrt{(160 \times 40)}}{10} = \frac{3}{5} \sqrt{(160 \times 40)} \\ 40) = \frac{3}{5} \sqrt{6400} &= \frac{3}{5} \times 80 = \frac{3 \times 80}{5} = 3 \times 16 = 48 = \text{ordinate FH.} \end{aligned}$$

2. The transverse diameter is 24, the conjugate 21, and the lesser abscissa 8: what is its ordinate?

Ans. 14.

3. The transverse diameter of a hyperbola is 50, the conjugate 30, and the less abscissa 12: required the ordinate.

CASE II.

The transverse and conjugate diameters, and an ordinate being given, to find the two abscissas.

RULE.

As the conjugate diameter is to the transverse,
 So is the square root of the sum of the squares of
 the ordinate and semi-conjugate,
 To the distance between the ordinate and the centre,
 or half the sum of the abscissas.

Then the sum of this distance and the semi-transverse will give the greater abscissa, and their difference the lesser abscissa required. .

EXAMPLES.

The transverse diameter is 120, the conjugate 72,
 and the ordinate 48: what are the two abscissas?

$1296 = \text{square of the semi-conjugate.}$

$2304 = \text{square of the ordinate.}$

$3600 (60 = \text{square root.})$

36

00

fe 72 : 120 :: 60

120

$72)7200 (100 = \frac{1}{2} \text{ sum of the abscissas.}$

72 60 = semi-transverse.

160 = greater abscissa.

40 = lesser abscissa.

2. The transverse and conjugate diameters are 24 and 21: required the two abscissas to the ordinate 14. *Ans. 32 and 8.*

3. The transverse being 60, and the conjugate 36: required the two abscissas to the ordinate 24. *Ans. 80 and 20.*

CASE III.

The transverse diameter, the two abscissas and the ordinate being given, to find the conjugate.

RULE.

As the square root of the product of the two abscissas,
Is to the ordinate;
So is the transverse diameter,
To the conjugate.

EXAMPLES.

1. The transverse diameter is 120, the ordinate is 48, and the two abscissas are 160 and 40: required the conjugate.

$$\begin{array}{r}
 160 \\
 40 \\
 \hline
 6400(80 \\
 64 \\
 \hline
 00
 \end{array}$$

K 2

As 80 : 48 :: 120 the transverse axis.

$$\begin{array}{r}
 48 \\
 \hline
 960 \\
 480 \\
 \hline
 8.0)576.0 \\
 \hline
 \end{array}$$

72 = conjugate required.

2. The transverse diameter is 24, the ordinate 14, and the abscissas 8 and 32 : required the conjugate.

Ans. 21.

CASE IV.

The conjugate diameter, the ordinate, and two abscissas being given, to find the transverse.

RULE.

1. Add the square of the ordinate to the square of the semi-conjugate, and find the square root of their sum.

2. Take the sum or difference of the semi-conjugate and this root, according as the greater or lesser abscissa is used, and then say,

As this sum or difference,

Is to the abscissa,

So is the conjugate,

To the transverse.

EXAMPLES.

1. The conjugate diameter is 72, the ordinate is 48, and the lesser abscissa 40 : what is the transverse?

Here $\sqrt{48^2 + 36^2} = \sqrt{2304 + 1296} = \sqrt{3600} = 60$, and $60 - 36 = 24$.

As $24 : 40 :: 72 : 120 = \text{transverse required}$.

2. The conjugate diameter is 21, the lesser abscissa 8, and its ordinate 14: required the transverse.

Ans. 24.

3. Required the transverse diameter of the hyperbola, whose conjugate is 36, the lesser abscissa being 20, and ordinate 24.

Ans. 60.

PROBLEM XIII.

To find the length of any arc of an hyperbola, beginning at the vertex.

RULE*.

1. To 19 times the transverse, add 21 times the parameter of the axis, and multiply the sum by the quotient of the abscissa divided by the transverse.

2. To 9 times the transverse, add 21 times the parameter; and multiply the sum by the quotient of the abscissa divided by the transverse.

3. To each of the products, thus found, add 15 times the parameter, and divide the former by the latter; then this quotient being multiplied by the ordinate will give the length of the arc *nearly*.

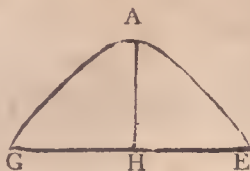
* If t = semi-transverse, c = semi-conjugate, and y = ordinate drawn from the end of the required arc; then $y \times$

$$\left(1 + \frac{t^2 y^2}{6c^4} A - \frac{t^2 + 4c^2}{c^2} \cdot \frac{3y^2}{20} B + \frac{t^4 + 4t^2 c^2 + 8c^4}{t^2 + 4c^2} \cdot \frac{5y^2}{14c^4} C \text{ \&c.} \right)$$

= length of the arc.

EXAMPLES.

1. In the hyperbola GAE, the transverse diameter is 80, the conjugate 60, the ordinate GH 10, and the abscissa AH 2.1637 : required the length of the arc AG.



$$\text{Here } 80 : 60 :: 60 : \frac{60 \times 60}{80} = \frac{3 \times 60}{4} = 3 \times 15 = 45$$

=parameter.

$$\text{And } 80 \times 19 + 45 \times 21 \times \frac{2.1637}{80} = 1520 + 945 \times .02704 = 2465 \times .02704 = 66.6536.$$

$$\text{Also } 80 \times 9 + 45 \times 21 \times \frac{2.1637}{80} = 720 + 6045 \times .02704 = 1665 \times .02704 = 45.0216.$$

Whence $67.5 + 66.6536 \div 67.5 \times 45.0216 = 741.6536 \div 720.0216 = 1.03004$; and $1.03004 \times 10 = 10.3004$ = length of the arc required.

2. The transverse diameter of an hyperbola is 120, the conjugate 72, the ordinate 48, and the abscissa 40 : required the length of the curve.

Ans 62.6496.

3. Required the length of the curve of an hyperbola, to the ordinate 10 ; the transverse and conjugate axes being 80 and 60.

Ans. 10.3.

PROBLEM XIV.

To find the area of an hyperbola, the transverse, conjugate, and abscissa being given.

RULE.

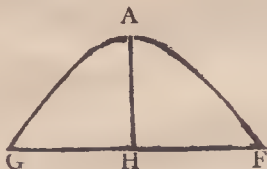
1. To the product of the transverse and abscissa, add $\frac{5}{7}$ of the square of the abscissa, and multiply the square root of the sum by 21.

2. Add 4 times the square root of the product of the transverse and abscissa, to the product last found, and divide the sum by 75.

3. Divide 4 times the product of the conjugate and abscissa by the transverse, and this last quotient multiplied by the former will give the area required nearly.

EXAMPLES.

In the hyperbola GAE, the transverse axis is 30, the conjugate 18, and the abscissa or height AH is 10 : what is the area ?



$$\begin{aligned} \text{Here } 21 \sqrt{30 \times 10 + \frac{5}{7} \times 10^2} &= 21 \sqrt{300 + 500 \div 7} \\ &= 21 \sqrt{300 + 71.42857} = 21 \sqrt{371.42857} = 21 \times \\ &19.272 = 404.712. \end{aligned}$$

$$\begin{aligned} \text{And } (4 \sqrt{30 \times 10} + 404.712) \div 75 &= (4 \sqrt{300} + \\ 404.712) \div 75 &= (4 \times 17.3205 + 404.712) \div 75 = \\ (69.282 + 404.712) \div 75 &= 473.994 \div 75 = 6.3199. \end{aligned}$$

Whence $\frac{18 \times 10 \times 4}{50} \times 6.3199 = \frac{18 \times 4}{3} \times 6.3199 =$
 $6 \times 4 \times 6.3199 = 24 \times 6.3199 = 151.6776 = \text{area re-}$
quired.

2. The transverse diameter is 100, the conjugate 60, and the lesser abscissa 50 : what is the area of the hyperbola ? *Ans.* 3220.363472.

3. Required the area of the hyperbola to the abscissa 25, the two axes being 50 and 30.

Ans. 805.0909.

OF THE
MENSURATION
OF
SOLIDS.

DEFINITIONS.

1. **T**HE *measure* of any solid body is the whole capacity or content of that body, when considered under the triple dimensions of length, breadth, and thickness.

2. A *cube* whose side is one inch, one foot, or one yard, &c. is called the *measuring unit*; and the content or solidity of any figure, is computed by the number of those cubes contained in that figure.

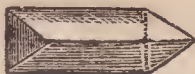
3. A *cube* is a solid contained by six equal square sides.



4. A *parallelopipedon* is a solid contained by six quadrilateral planes, every opposite two of which are equal and parallel.



5. A *prism* is a solid whose ends are two equal, parallel, and similar plane figures, and its sides parallelograms.



6. A *cylinder* is a solid described by the revolution of a right-angled parallelogram about one of its sides, which remains fixed.



7. A * *pyramid* is a solid whose sides are all triangles meeting in a point at the vertex, and the base any plane figure whatever.



8. A *sphere* is a solid described by the revolution of a semicircle about its diameter, which remains fixed.



* The definition of a cone has been given already.

9. The centre of a sphere is a point within the figure, everywhere equally distant from the convex surface of it.

10. The diameter of the sphere is a straight line passing through the centre, and terminated both ways by the convex superficies.

11. A *circular spindle* is a solid generated by the revolution of a segment of a circle about its chord, which remains fixed.



12. A *spheroid* is a solid generated by the revolution of a semi-ellipsis about one of its diameters, which is considered as quiescent.

The spheroid is called *prolate*, when the revolution is made about the transverse diameter, and *oblate* when it is made about the conjugate diameter.



13. *Elliptic, parabolic, and hyperbolic spindles* are generated in the same manner as the circular spindle, the double ordinate of the section being always fixed or quiescent.

14. *Parabolic and hyperbolic conoids* are solids formed by the revolution of a semi-parabola or hyperbola about its transverse axis, which is considered as quiescent.



15. The *segment* of a pyramid, sphere, or any other solid, is a part cut off from the top by a plane parallel to the base of that figure.

16. A *frustum* or *trunk* is the part that remains at the bottom after the segment is cut off.

17. The *zone of a sphere* is that part which is intercepted between two parallel planes; and when those planes are equally distant from the centre, it is called the middle zone of the sphere.

18. The height of a solid is a perpendicular, drawn from its vertex to the base or plane on which it is supposed to stand.

PROBLEM I.

To find the solidity of a cube, the height of one of its sides being given.

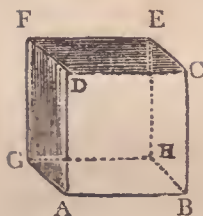
RULE*.

Multiply the side of the cube by itself, and that product again by the side, and it will give the solidity required.

EXAMPLES.

8. The side AB, or BC of the cube ABCDFGHE, is 25.5: what is the solidity?

* *Note.* The surface of the cube is equal to six times the square of its side.



Here $AB^3 = \overline{25.5}^3 = 25.5 \times 25.5 \times 25.5 = 25.5 \times 650.25 = 16581.375$, the answer, or content of the cube.

2. The side of a cube is 15 inches: what is the solidity?

fo. in. fa.

Ans. 1 11 5

3. What is the solidity of a cube whose side is 17.5 inches?

Ans. 3.101 feet.

PROBLEM II.

To find the solidity of a parallelopipedon.

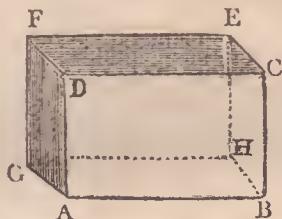
RULE*.

Multiply the length by the breadth, and that product again by the depth or altitude, and it will give the solidity required.

* *Note.* The surface of the parallelopipedon is equal to the sum of the areas of all its sides or ends.

EXAMPLES.

1. Required the solidity of the parallelopipedon ABCDFEGH, whose length AB is 8 feet, its breadth FD $4\frac{1}{2}$ feet, and the depth or altitude AD $6\frac{3}{4}$ feet.



Here $AB \times AD \times FD = 8 \times 6.75 \times 4.5 = 54 \times 4.5 = 243$ solid feet, the contents of the parallelopipedon required.

2. The length of a parallelopipedon is 15 feet, and each side of its square base 21 inches: what is the solidity? *Ans. 45.9 feet.*

3. What is the solidity of a block of marble, whose length is 10 feet, its breadth $5\frac{3}{4}$ feet, and the depth $3\frac{1}{2}$ feet? *Ans. 201.25 feet.*

PROBLEM III.

To find the solidity of a prism.

RULE*.

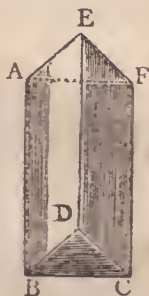
Multiply the area of the base into the perpendicular

* The surface of a prism is equal to the sum of the areas of the two ends and of all its sides.

lar height of the prism, and the product will be the solidity.

EXAMPLES.

1. What is the solidity of the triangular prism ABCFED, whose length AB is 10 feet, and either of the equal sides BC, CD, or DB, of one of its equilateral ends BCD, $2\frac{1}{2}$ feet?



* Here $\frac{1}{4} \times 2.5^2 \times \sqrt{3} = \frac{1}{4} \times 6.25 \times \sqrt{3} = 1.5625 \times \sqrt{3} = 1.5625 \times 1.732 = 2.70625 = \text{area of the base BCD.}$

Or $\frac{2.5 + 2.5 + 2.5}{2} = \frac{7.5}{2} = 3.75 = \frac{1}{2} \text{ sum of the sides BC, CD, DB of the triangle CDB.}$

And $3.75 - 2.5 = 1.25$, $\therefore 1.25, 1.25$, and $1.25 = 3$ differences.

Whence $\sqrt{3.75 \times 1.25 \times 1.25 \times 1.25} = \sqrt{3.75 \times 1.25^3} = \sqrt{7.32521875} = 2.7063 = \text{area of the base as before.}$

* This operation is agreeable to the rule problem 8, page 65, the number $.433013 = \frac{1}{4}\sqrt{3}$.

And $2.7063 \times 10 = 27.063$ solid feet, the content of the prism required.

2. What is the solidity of a triangular prism, whose length is 18 feet, and one side of the equilateral end $1\frac{1}{2}$ feet? *Ans. 17.50859 feet.*

2. Required the solidity of a prism whose base is a hexagon, supposing each of the equal sides to be 1 foot 4 inches, and the length of the prism 15 feet. *Ans. 69.282 feet.*

PROBLEM IV.

To find the convex surface of a cylinder.

RULE*.

Multiply the periphery or circumference of the base by the height of the cylinder, and the product will be the convex surface required.

EXAMPLES.

1. What is the convex surface of the right cylinder ABCD, whose length BC is 20 feet, and the diameter of its base AB 2 feet?

* *Note.* If twice the area of either of the ends be added to the convex surface, it will give the whole surface of the cylinder



Here $3.14159 \times 2 = 6.28318 =$ periphery of the base AB.

And $6.28318 \times 20 = 125.6636$ square feet, the convexity required.

2. What is the convex surface of a right cylinder, the diameter of whose base is 30 inches, and the length 60 inches? *Ans. 5654.862 inches.*

3. Required the convex superficies of a right cylinder, whose circumference is 8 feet 4 inches, and its length 14 feet. *Ans. 116.6655, &c. feet.*

PROBLEM V.

To find the solidity of a cylinder.

RULE*.

Multiply the area of the base by the perpendicular height of the cylinder, and the product will be the solidity.

* The four following cases contain all the rules for finding the superficies and solidities of *cylindric ungulas*.

I. *When the section is parallel to the axis of the cylinder.*

EXAMPLES.

1. What is the solidity of the cylinder ABCD, the diameter of whose base AB is 30 inches, and the height BC 50 inches?



Rule. 1. Multiply the length of the arc line of the base by the height of the cylinder, and the product will be the *curve surface*.

2. Multiply the area of the base by the height of the cylinder, and the product will be the *solidity*.

II. When the section passes obliquely through the opposite sides of the cylinder.



Rule. 1. Multiply the circumference of the base of the cylinder by half the sum of the greatest and least length of the ungula, and the product will be the *curve surface*.

2. Multiply the area of the base of the cylinder by half the sum of the greatest and least lengths of the ungula, and the product will be the *solidity*.

III. When the section passes through the base of the cylinder, and one of its sides.



Here $.7854 \times 30^2 = .7854 \times 900 = 706.86 = \text{area of the base AB.}$

And $706.86 \times 50 = 35343 \text{ cubic inches; or } \frac{35343}{1728} = 20.4531 \text{ solid feet, the answer required.}$



Rule. 1. Multiply the sine of half the arc of the base by the diameter of the cylinder, and from this product subtract the product of the arc and cosine.

2. Multiply the difference, thus found, by the quotient of the height divided by the versed sine, and the product will be the *curve surface*.

1. From two-thirds of the cube of the right sine of half the arc of the base, subtract the product of the area of the base and the cosine of the said half arc.

2. Multiply the difference, thus found, by the quotient arising from the height divided by the versed sine, and the product will be the *solidity*.

IV. *When the section passes obliquely through both ends of the cylinder.*

2. What is the solidity of a cylinder whose height is 5 feet, and the diameter of the end 2 feet?

Ans. 15.708 feet.

3. What is the solidity of a cylinder whose height is 20 feet, and the circumference of its base 20 feet also?

Ans. 636.64 feet.

4. The circumference of the base of an oblique cylinder is 20 feet, and the perpendicular height 19.318: what is the solidity?

Ans. 614.926 feet.

PROBLEM VI.

To find the convex surface of a right cone.



Rule. 1. Conceive the section to be continued, till it meets the side of the cylinder produced; then say, as the difference of the versed sines of half the arcs of the two ends of the ungula, is to the versed sine of half the arc of the lesser end, so is the height of the cylinder to the part of the side produced.

2. Find the surface of each of the ungulas, thus formed, by case the third, and their difference will be the *surface required*.

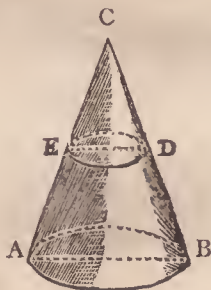
3. In like manner find the solidities of each of the ungulas, and their difference will be the *solidity required*.

RULE*.

Multiply the circumference of the base by the slant height, or the length of the side of the cone, and half the product will be the surface required.

EXAMPLES.

1. The diameter of the base AB is 3 feet, and the slant height AC or BC 15 feet: required the convex surface of the cone ACB.



Here $3.1416 \times 3 = 9.4248 =$ circumference of the base AB.

And $\frac{9.4248 \times 15}{2} = \frac{141.3720}{2} = 70.686$ square feet,
the convex surface required.

* To find the surface of a right pyramid.

Rule. Multiply the perimeter of the base by the length of the side, or slant height of the cone, and half the product will be the surface required, exclusive of the base.

2. The diameter of a right cone is 4.5 feet, and the slant height 20 feet: required the convex surface.

Ans. 141.372 feet.

3. The circumference of the base is 10.75, and the slant height 18.25: what is the convex surface?

Ans. 98.09375.

PROBLEM VII.

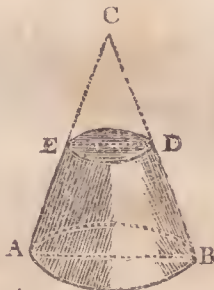
To find the convex surface of the frustum of a right cone.

RULE*.

Multiply the sum of the perimeters of the two ends by the slant height of the frustum, and half the product will be the surface required.

EXAMPLES.

1. In the frustum ABED, the circumferences of the two ends AB and DE are 22.5 and 15.75 respectively, and the slant height BD is 26: what is the convex surface?



* *To find the surface of the frustum of a right pyramid.*

Rule. Multiply the sum of the perimeters of the end by the slant height, and half the product will be the surface required, exclusive of the ends.

Here $\frac{(22.5 + 15.75) \times 26}{2} = 22.5 + 15.75 \times 13 =$
 $38.25 \times 15 = 497.25 = \text{convex surface required.}$

2 What is the convex surface of the frustum of a right cone, the circumference of the greater end being 30 feet, that of the lesser end 10 feet, and the length of the slant side 20 feet? *Ans. 400 feet.*

3. What is the convex surface of the frustum of a right cone, the diameters of the ends being 8 and 4 feet, and the length of the slant side 20 feet?

Ans. 376.992 feet.

4. If a segment of 6 feet slant height be cut off the top of a cone whose slant height is 30 feet, and circumference of its base 10 feet: what is the surface of the frustum?

Ans. 144 feet.

PROBLEM VIII.

To find the solidity of a cone or pyramid.

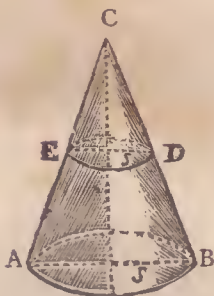
RULE.

Multiply the area of the base by $\frac{1}{3}$ of the perpendicular height of the cone, and the product will be the solidity.

EXAMPLES.

1. Required the solidity of the cone ACB, whose diameter AB is 20, and its perpendicular height CS 24.

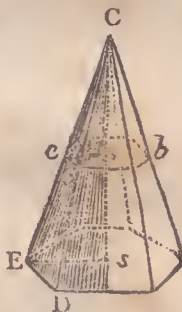
M



Here $.7854 \times 20^2 = .7854 \times 400 = 314.16 = \text{area of the base AB.}$

And $314.16 \times \frac{24}{3} = 314.16 \times 8 = 2513.28 = \text{solidity required.}$

2. Required the solidity of the hexagonal pyramid ECBD, each of the equal sides of its base being 40, and the perpendicular height CS 60.



Here 2.598076 (multiplier when the side is 1) $\times 40^2 = 2.598076 \times 1600 = 4156.9216 = \text{area of the base.}$

And $4156.9216 \times \frac{60}{3} = 4156.9216 \times 20 = 83138.432$,
solidity required.

3. Required the solidity of a triangular pyramid, whose height is 30, and each side of the base 3.

Ans. 38.97117.

4. Required the solidity of a square pyramid, each side of whose base is 30, and the perpendicular height 20.

Ans. 6000.

5. What is the solidity of a cone, the diameter of whose base is 18 inches, and its altitude 15 feet?

Ans. 8.83575 feet.

6. If the circumference of the base of a cone be 40 feet, and the height 50 feet: what is the solidity?

Ans. 2120 feet.

7. What is the content of a pentagonal pyramid, its height being 12 feet, and each side of its base 2 feet?

Ans. 27.527.

PROBLEM IX.

To find the solidity of a frustum of a cone or pyramid.

RULE*.

1. *For the frustum of a cone, the diameters of the two ends, and the height being given.*

* When the base is a regular polygon, add the square of a side of the greater end, the square of a side of the less, and the rectangle of those sides together, multiply by the proper tabular number (prob. 8, sup.), and again by one-third of the altitude.

Divide the difference of the cubes of the diameters of the two ends, by the difference of the diameters, and this quotient being multiplied by .7854 and again by $\frac{1}{3}$ of the height will give the solidity.

2. *For the frustum of a pyramid, the sides and height being given.*

To the areas of the two ends of the frustum add the square root of their product, and this sum being multiplied by $\frac{1}{3}$ of the height will give the solidity*.

* The following cases contain all the rules for finding the superficies and solidities of conical ungulas.

1. *When the section passes through the opposite extremities of the ends of the frustum.*

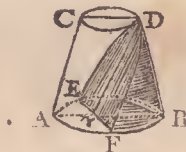


Let $D=AB$, the diameter of the greater end, $d=CD$ the diameter of the less end, h = perpendicular height of the frustum, and $n=.78539$, &c.

Then will $\frac{D^2-d^2}{D-d} \sqrt{Dd} \div D-d \times \frac{1}{3} n D h$ = solidity of the elliptic ungula ADB.

And $\frac{n}{D-d} \sqrt{4h^2 + (D-d)^2} \times \frac{D^2 - (D-d)^2}{D-d} \sqrt{4Dd}$ = curve surface of ADB.

2. *When the section cuts off part of the base, and makes the angle DrB less than the angle CAB.*



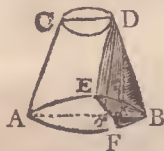
1. What is the solidity of the frustum of the cone EABD, the diameter of whose greater end AB is 5

Let S = tabular segment, whose versed sine is $Br \div D$,
 s = tab. seg. whose versed sine is $Br - (D - d) \div d$, and the
 other letters as before.

Then $(S \times D^3 - s \times d^3 \times \frac{Br}{Br - D - d} \sqrt{\frac{Br}{Br - D - d}}) \times \frac{\frac{1}{3}h}{D - d}$
 = solidity of the elliptic hoof EFBD.

And $\frac{1}{D - d} \sqrt{4h^2 + (D - d)^2} \times (\text{seg. FBE} - \frac{d^2}{D^2} \times$
 $\frac{\frac{1}{2} \times (D + d) - Ar}{d - Ar} \times \sqrt{\frac{Br}{d - Ar}} \times \text{seg. of the circle AB,}$
 whose height is $D \times \frac{d - Ar}{d}$ = convex surface of EFBD.

3. When the section is parallel to one of the sides of the frustum.



Let A = area of the base FBE, and the other letters as before.

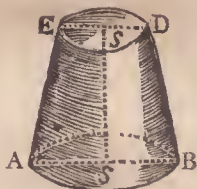
Then $(\frac{A \times D}{D - d} - \frac{4}{3}d \sqrt{(D - d) \times d}) \times \frac{1}{3}h$ = solidity of the parabolic hoof EFBD.

And $\frac{1}{D - d} \sqrt{4h^2 + (D - d)^2} \times (\text{seg. FBE} - \frac{2}{3} \frac{d^2}{D - d})$

$\sqrt{d \times D - d}$ = convex surface of EFBD.

4. When the section cuts off part of the base, and makes the angle DrB greater than the angle CAB .

feet, that of the lesser end ED 3 feet, and the perpendicular height sS 9 feet?



Let the area of the hyperbolic section $EDF = A$, and the area of the circular seg. $EBF = a$.

Then $\frac{\frac{1}{8}h}{D-d} \times (a \times D - A \times \frac{d \times Br}{Cr}) =$ solidity of the hyperbolic ungula $EFBD$.

And $\frac{1}{D-d} \times \sqrt{4h^2 + (D-d)^2} \times (\text{cir. seg. } EBF - \frac{d^2}{D^2} \times \frac{Br - \frac{1}{2}(D-d)}{Br - D - d}) \sqrt{\frac{Br}{Br - D - d}} =$ curve surface of $EFBD$.

Note. The transverse diameter of the hyp. seg. $= \frac{d \times Cr}{D - d - Br}$

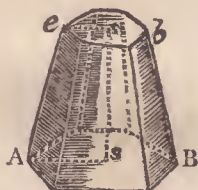
and the conjugate $= d \sqrt{\frac{Br}{D - d - Br}}$, from which its area may be found by the former rules.

$$\text{Here } \frac{5^3 - 3^3}{5 - 3} \times .7854 \times \frac{2}{3} = \frac{125 - 27}{2} \times .7854 \times 3 =$$

$$\frac{98}{2} \times 2.3562 = 49 \times 2.3562 = 115.4538 \text{ solid feet, the}$$

content of the frustum.

2. What is the solidity of the frustum $eEDBb$, of an hexagonal pyramid, the side ED of whose greater end is 4 feet, that ed of the lesser end 3 feet, and the height Ss 9 feet?



Here $2.598076 \text{ (tab. mult.)} \times 3^2 = 2.598076 \times 9 = 23.382624 = \text{area of the polygon } eb.$

And $2.598076 \text{ (tab. mult.)} \times 4^2 = 2.598076 \times 16 = 41.569216 = \text{area of the polygon } EB.$

Whence $\sqrt{41.569216 \times 23.382684} = \sqrt{971.999841} = 31.176911.$

And $(41.569216 + 23.382684 + 31.176911) \times \frac{2}{3} = 96.128811 \times 3 = 288.386433 \text{ feet} = \text{solidity required.}$

Or, thus, $(4^2 + 3^2 + 4 + 3) \times 2.598076 \times \frac{9}{3} = 37 \times 2.598076 \times 3 = 288.386436 \text{ Ans.}$

3. What is the solidity of the frustum of a cone, the diameter of the greater end being 4 feet, that of the lesser end 2 feet, and the altitude 9 feet?

Ans. 65.9736.

4. What is the solidity of the frustum of a cone, the circumference of the greater end being 40, that of the lesser end 20, and the length or height 50?

Ans. 3713.64.

5. What is the solidity of the frustum of a square pyramid, one side of the greater end being 18 inches, that of the lesser end 15 inches, and the height 60 inches?

Ans. 16380 inches.

6. What is the solidity of the frustum of an hexagonal pyramid, the side of whose greater end is 3 feet, that of the lesser end 2 feet, and the length 12 feet?

Ans. 197.453776 feet.

PROBLEM X.

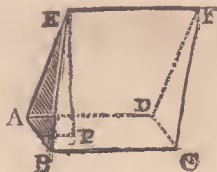
To find the solidity of a cuneus or wedge.

RULE.

Multiply the sum of twice the length of the base and the length of the edge, by the product of the height of the wedge and the breadth of the base, and $\frac{1}{6}$ of the last product will be the solidity.

EXAMPLES.

1. How many solid feet are there in a wedge, whose base is 5 feet 4 inches long, and 9 inches broad, the length of the edge being 3 feet 6 inches, and the perpendicular height 2 feet 4 inches?



$$\begin{aligned} \text{Here } \frac{(64 \times 2 + 42) \times 28 \times 9}{6} &= \frac{(128 + 42) \times 28 \times 9}{6} \\ &= \frac{170 \times 28 \times 9}{6} = \frac{170 \times 28 \times 3}{2} = 170 \times 14 \times 3 = 7140 \end{aligned}$$

solid inches.

And $7140 \div 1728 = 4.1319$ solid feet, the content required.

2. The length and breadth of the base of a wedge are 35 and 15 inches, and the length of the edge is 55 inches : what is the solidity, supposing the perpendicular height to be 17.14508 inches ?

Ans. 3.1006 feet.

PROBLEM XI.

To find the solidity of a prismoid.

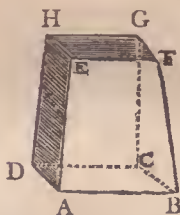
RULE.

To the sum of the areas of the two ends add four times the area of a section parallel to and equally distant from both ends, and this last sum multiplied by $\frac{1}{6}$ of the height will give the solidity.

Note. The length of the middle rectangle is equal to half the sum of the lengths of the rectangles of the two ends, and its breadth equal to half the sum of the breadths of those rectangles.

EXAMPLES.

1. What is the solidity of a rectangular prismoid, the length and breadth of one end being 14 and 12 inches, and the corresponding sides of the other 6 and 4 inches ; and the perpendicular $30\frac{1}{2}$ feet ?



Here $14 \times 12 + 6 \times 4 = 168 + 24 = 192 = \text{sum of the areas of the two ends.}$

Also $\frac{14+6}{2} = \frac{20}{2} = 10 = \text{length of the middle rectangle.}$

And $\frac{12+4}{2} = \frac{16}{2} = 8 = \text{breadth of the middle rectangle.}$

Whence $10 \times 8 \times 4 = 80 \times 4 = 320 = 4 \text{ times the area of the middle rectangle.}$

Or $(320 + 192) \times \frac{366}{6} = 512 \times 61 = 31232 \text{ solid inches.}$

And $31232 \div 1728 = 18.074 \text{ solid feet, the content.}$

2. What is the solid content of a prismoid, whose greater end measures 12 inches by 8, the lesser end 8 inches by 6, and the length, or height, 60 inches?

Ans. 2.453 feet.

3. What is the capacity of a coal-waggon, whose inside dimensions are as follow: at the top, the length is $81\frac{1}{2}$, and breadth 55 inches; at the bottom, the length is 41, and the breadth $29\frac{1}{2}$ inches; and the perpendicular depth is $47\frac{1}{4}$ inches?

Ans. 126340.59375 cubic inches; which is nearly equal to a chaldron of coals.

PROBLEM XII.

To find the convex surface of a sphere.

RULE*.

Multiply the diameter of the sphere by its circumference, and the product will be the convex superficies required.

Note. The curve surface of any zone or segment will also be found by multiplying its height by the whole circumference of the sphere.

EXAMPLES.

1. What is the convex superficies of a globe BC, whose diameter BG is 17 inches?
-

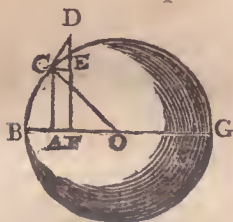
* 1. To find the lunar surface included between two great circles of the sphere.

RULE. Multiply the diameter into the breadth of the surface in the middle, and the product will be the superficies required.

OR, As one right angle is to a great circle of the sphere;
So is the angle made by the two great circles,
To the surface included by them.

2. To find the area of a spherical triangle, or the surface included by the intersecting arcs of three great circles of the sphere.

RULE. As two right angles, or 180° ,
Is to a great circle of the sphere;
So is the excess of the three angles above two right angles,
To the area of the triangle.



Here $3.14159 \times 17 \times 17 = 53.40703 \times 17 = 907.91951$ square inches.

And $907.91951 \div 144 = 6.30499$ square feet, the answer.

2. What is the convex superficies of a sphere whose diameter is $1\frac{1}{3}$ feet, and the circumference 4.1888 feet? *Ans. 5.58506 feet.*

3. If the diameter or axis of the earth be $7957\frac{3}{4}$ miles, what is the whole surface, supposing it to be a perfect sphere? *Ans. 198943653 square miles.*

4. The diameter of a sphere is 21 inches; what is the convex superficies of that segment of it whose height is $4\frac{1}{2}$ inches? *Ans. 296.8802 inches.*

5. What is the convex surface of a spherical zone, whose breadth is 4 inches, and the diameter of the sphere from which it was cut 25 inches?

Ans. 314.16 inches.

PROBLEM XIII.

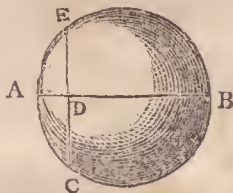
To find the solidity of a sphere or globe.

RULE.

Multiply the cube of the diameter by .5236, and the product will be the solidity.

EXAMPLES.

1. What is the solidity of the sphere ACBE, whose diameter AB is 17 inches?



Here $17^3 \times .5236 = 17 \times 17 \times 17 \times .5236 = 289 \times 17 \times .5236 = 4913 \times .5236 = 5272.4468$ solid inches;

And $5272.4468 \div 1728 = 1.48868$ solid feet, the answer.

2. What is the solidity of a sphere whose diameter is $1\frac{1}{3}$ feet? *Ans. 1.2411 feet.*

3. What is the solidity of the earth, supposing it to be perfectly spherical, and its diameter $7957\frac{3}{4}$ miles? *Ans. 263,858,149,120 miles.*

PROBLEM XIV.

To find the solidity of the segment of a sphere.

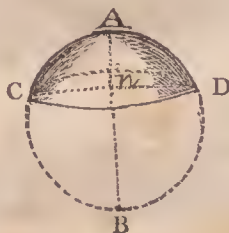
RULE.

To three times the square of the radius of its base add the square of its height; and this sum multiplied by the height, and the product again by .5236 will give the solidity.

Or, From three times the diameter of the sphere subtract twice the height of the segment, multiply by the square of the height, and that product by .5236; the last product will be the solidity.

EXAMPLES.

1. The radius Cn of the base of the segment CAD is 7 inches, and the height An 4 inches: what is its solidity?



Here $(7^2 \times 3 + 4^2) \times 4 \times .5236 = (49 \times 3 + 4^2) \times 4 \times .5236 = (147 + 4^2) \times 4 \times .5236 = (147 + 16) \times 4 \times .5236 = 163 \times 4 \times .5236 = 652 \times .5236 = 341.3872$ solid inches, the answer.

2. What is the solidity of the segment of a sphere, the diameter of whose base is 20, and its height 9?

Ans. 1799.6132.

3. What is the content of the spherical segment, whose height is 4 inches, and the radius of its base 8?

Ans. 435 6352.

4. What is the solidity of a spherical segment, the diameter of its base being 17.23368, and its height 4.5?

Ans. 572.5566.

5. The diameter of a sphere being 6 inches, required the solidity of the segment whose altitude is 2 inches.

Ans. 29.3216.

6. Required the solidity of a spherical segment, the height of which is 15, the diameter of the sphere being 18. *Ans.* 2827.44.

PROBLEM XV.

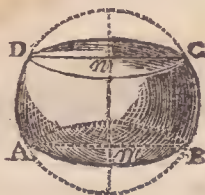
To find the solidity of a frustum or zone of a sphere.

RULE*.

To the sum of the squares of the radii of the two ends, add $\frac{1}{3}$ of the square of their distance, or the breadth of the zone, and this sum multiplied by the said breadth, and the product again by 1.5708, will give the solidity.

EXAMPLES.

1. What is the solid content of the zone ABCD, whose greater diameter AB is 20 inches, the lesser diameter CD 15 inches, and the distance *mn* of the two ends 10 inches?



* If it be the middle zone of the sphere, the solidity will be $= (a^2 + \frac{2}{3}h^2) \times .7854h$; where d = diameter of each end, and h = its height.

Here $(10^2 + 7.5^2 + \frac{10^2}{3}) \times 10 \times 1.5708 = (100 + 56.25 + 33.33) \times 10 \times 1.5708 = 189.58 \times 10 \times 1.5708 = 1895.8 \times 1.5708 = 2977.92264$ solid inches, the answer.

2. What is the solid content of a zone, whose greater diameter is 24 inches, the lesser diameter 20 inches, and the distance of the ends 4 inches?

Ans. 1566.6112 inches.

3. Required the solidity of the middle zone of a sphere, whose top and bottom diameters are each 3 feet, and the breadth of the zone 4 feet.

Ans. 61.7848 feet.

PROBLEM XVII.

To find the surface of a circular spindle, the length and breadth, or middle diameter, being given.

RULE.

1. To the square of half the length of the spindle, or longest diameter, add the square of half the middle diameter, and this sum, divided by the middle diameter, will give the *radius of the circle*.

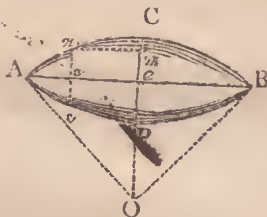
2. Take half the middle diameter from the radius of the circle, and it will give the *central distance*.

3. Find the length of the revolving arc by problem the 10th of Superficies.

4. From the product of the longest diameter and the radius of the revolving arc, subtract the product of the said arc and the central distance, and this remainder, multiplied by 6.2832, will give the surface required.

EXAMPLES.

1. What is the superficial content of the circular spindle ADBC, whose length AB is 48, and the middle diameter CD 36?



$$\text{Here } (24^2 + 18^2) \div 36 = (576 + 324) \div 36 = \frac{900}{36} =$$

$$\frac{100}{4} = 25 = OC, \text{ the radius of the circle } ACB. \quad i$$

Therefore $25 - 18 = 7 = OC - Ce = \text{central distance } Oe.$

$$\text{And } \frac{2 \times 18}{3} \div (50 - \frac{41 \times 18}{50}) = 12 \div 50 = 14.76 =$$

$$12 \div 35.24 = .34052.$$

Whence $(1 + .34052) \times 48 = 1.34052 \times 48 = 64.34496 = \text{length of the arc } ACB.$

And $(48 \times 25 - 64.34496 \times 7) \times 6.2832 = (1200 - 450.41472) \times 6.2832 = 749.58528 \times 6.2832 = 4709.794231296 = \text{superficies required.}$

2. What is the superficial content of a circular spindle whose length is 48, and its middle diameter 30?

Ans. 4387.1644.

PROBLEM XVIII.

To find the solidity of a circular spindle, the length and middle diameter being given.

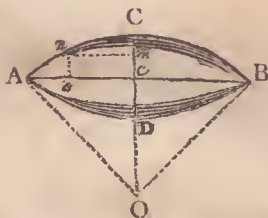
RULE.

1. Find the area of the generating circular segment by problem the 13th; and the radius and central distance as in the last problem.

2. From $\frac{1}{3}$ of the cube of half the length of the spindle subtract the product of the central distance and half the area of the generating segment, and this remainder multiplied by 12.5664 will give the solidity.

EXAMPLES.

1. The longest diameter AB of the circular spindle ADBC is 48, and the middle diameter CD 36: what is the solidity of the spindle?



Here $\frac{48 \times 7}{2} = 24 \times 7$ (7 being the central distance Oe, as in the last prob.) = 168 = area of the triangle ACB.

And 32.17248 ($\frac{1}{2}$ length of the arc ACB by last prob.) $\times 25$ (radius OC by same prob.) $= 804.312$
 $=$ area of the sector B CAO.

Also $804.312 - 168 = 636.312 =$ area of the segment ACBe.

And $(\frac{24^3}{3} - \frac{636.312}{2} \times 7) \times 12.5664 = (\frac{13824}{3} - 318.156 \times 7) \times 12.5664 = (4608 - 2227.092) \times 12.5664 = 2380.908 \times 12.5664 = 29919.4422912 =$
 solidity required.

2. If the length of a circular spindle be 40, and its middle diameter 30 : what is its solidity ?

Ans. 17310.858.

PROBLEM XIX.

To find the solidity of the middle frustum of a circular spindle, the length of the frustum, the middle diameter, and that of either of the ends being given.

RULE.

1. Divide the square of half the length of the frustum by half the difference of the middle diameter, and that of either of the two ends; and half this quotient added to $\frac{1}{4}$ of the said difference will give the radius of the circle.

2. Find the central distance, and the revolving area, as in the last problem.

3. From the square of the radius take the square of the central distance, and the square root of the remainder will give half the length of the spindle.

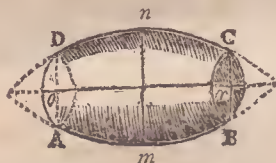
4. From the square of half the length of the spindle take $\frac{1}{3}$ of the square of half the length of the

frustum, and multiply the remainder into the said half length.

5. From this product take that of the generating area and central distance, and the remainder multiplied by 6.2832 will give the content of the frustum.

EXAMPLES.

1. What is the solidity of the frustum ABCD, whose middle diameter nm is 36, the diameter DA or CB 16, and the length or 40?



Here $\frac{1}{2} \times (20^2 \div \frac{36-16}{2}) + \frac{36-16}{4} = \frac{1}{2} \times (400 \div 10) + 5 = \frac{1}{2} \times 40 + 5 = 20 + 5 = 25 = \text{radius of the circle.}$

Consequently $25 - \frac{1}{2}nm = 25 - 18 = 7 = \text{central distance.}$

And $\frac{10}{50} = \frac{1}{5} = .2 = \text{tab. versed sine; and .111823} = \text{tab. segment.}$

Also $.111823 \times 50^2 = .111823 \times 2500 = 279.5575 = \text{area of the segment DnC.}$

And $279.5575 + 320 = 599.5575 = \text{generating area ODnCr.}$

Again $\sqrt{(25^2 - 7^2)} = \sqrt{(625 - 49)} = \sqrt{576} = 24 = \frac{1}{2} \text{ length of the spindle.}$

$$\begin{aligned}
 &\text{And } (24^2 - \frac{20^2}{3}) \times 20 - 599.5575 \times 7) \times 6.2832 = \\
 &(556 - 133.3) \times 20 - 4196.9025) \times 6.2832 = \\
 &(8853.334 - 4196.9025) \times 6.2832 = 4656.4135 \times \\
 &6.2832 = 29257.2904 \text{ solidity required.}
 \end{aligned}$$

The middle diameter of the frustum of a circular spindle is 32, the diameter at the end 24, and the length 40: what is the solidity?

Ans. 27287.5411256 cubic inches.

PROBLEM XX.

To find the solidity of a spheroid.

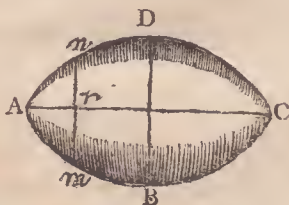
RULE.

Multiply the square of the revolving axe by the fixed axe, and this product again by .5236, and it will give the solidity required.

Where note that .5236 is $= \frac{1}{6}$ of 3.14159.

EXAMPLES.

1. In the prolate spheroid ABCD, the transverse or fixed axe AC is 90, and the conjugate or revolving axe DB is 70: what is the solidity?



Here $DB^2 \times AC \times .5236 = 70^2 \times 90 \times .5236 = 4900 \times 90 \times .5236 = 441000 \times .5236 = 230907.6 =$
solidity required.

2. What is the solidity of a prolate spheroid, whose fixed axe is 100, and its revolving axe 60?

Ans. 188400.

3. What is the solidity of an oblate spheroid, whose fixed axe is 60, and its revolving axe 100?

Ans. 314160.

PROBLEM XXI.

To find the content of the middle frustum of a spheroid, its length, the middle diameter, and that of either of the ends being given.

CASE I.

When the ends are circular, or parallel to the revolving axis.

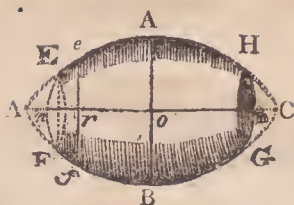
RULE.

To twice the square of the middle diameter add the square of the diameter of either of the ends, and this sum multiplied by the length of the frustum, and the product again by .2618, will give the solidity.

Where note that $.2618 = \frac{1}{12}$ of 3.14159.

EXAMPLES.

1. In the middle frustum of a spheroid EFGH, the middle diameter DB is 50 inches, and that of either of the ends EF or GH 40 inches, and its length *nm* 18 inches: what is its solidity?



Here $(50^2 \times 2 + 40^2) \times 18 \times .2618 = (2500 \times 2 + 1600) \times 18 \times .2618 = (5000 + 1600) \times 18 \times .2618 = 6600 \times 18 \times .2618 = 118800 \times .2618 = 31101.84$ cubic inches, the answer.

2. What is the solidity of the middle frustum of a prolate spheroid, the middle diameter being 60, that of either of the two ends 36, and the distance of the ends 80? *Ans.* 177940.224.

3. What is the solidity of the middle frustum of an oblate spheroid, the middle diameter being 100, that of either of the ends 80, and the distance of the ends 36? *Ans.* 248814.72.

CASE II.

When the ends are elliptical or perpendicular to the revolving axis.

RULE.

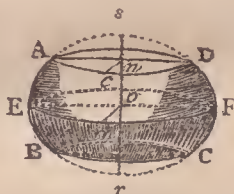
1. Multiply twice the transverse diameter of the middle section by its conjugate diameter, and to this product add the product of the transverse and conjugate diameters of either of the ends.

2. Multiply the sum, thus found, by the distance of the ends, or the height of the frustum, and the

product again by .2618, and it will give the solidity required.

EXAMPLES.

1. In the middle frustum ABCD of an oblate spheroid, the diameters of the middle section EF are 50 and 30; those of the end AD 40 and 24; and its height *nc* 18: what is the solidity?



Here $(50 \times 2 \times 30 + 40 \times 24) \times 18 \times .2618 = (3000 + 960) \times 18 \times .2618 = 3960 \times 18 \times .2618 = 71280 \times .2618 = 18661.104 = \text{solidity required.}$

2. In the middle frustum of a prolate spheroid, the diameters of the middle section are 100 and 60; those of the end 80 and 48; and the length 36: what is the solidity? *Ans.* 149288.832.

3. In the middle frustum of an oblate spheroid, the diameters of the middle section are 100 and 60; those of the end 60 and 36; and the length 80: what is the solidity of the frustum? *Ans.* 296567.04.

PROBLEM XXII.

To find the solidity of the segment of a spheroid.

CASE I.

When the base is parallel to the revolving axis.

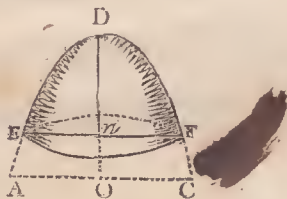
RULE.

1. Divide the square of the revolving axis by the square of the fixed axe, and multiply the quotient by the difference between three times the fixed axe and twice the height of the segment.

2. Multiply the product, thus found, by the square of the height of the segment, and this product again by .5236, and it will give the solidity required.

EXAMPLES.

1. In the prolate spheroid DEFD, the transverse axis 2DO is 100, the conjugate AC 60, and the height Dn of the segment EOF 10: what is the solidity?



$$\text{Here } \left(\frac{60^2}{100^2} \times 300 - 10 \right) \times 10^2 \times .5236 = .36 \times$$

$$280 \times 10^2 \times .5236 = 100.80 \times 100 \times .5236 = 10080 \times$$

$$.5236 = 5277.888 = \text{solidity required.}$$

2. The axes of a prolate spheroid are 50 and 30: what is the solidity of that segment whose height is 5, and its base perpendicular to the fixed axe?

Ans. 659.746

3. The diameters of an oblate spheroid are 100 and 60: what is the solidity of that segment whose height is 12, and its base perpendicular to the conjugate axe? *Ans.* 32672.64.

CASE II.

When the base is perpendicular to the revolving axis.

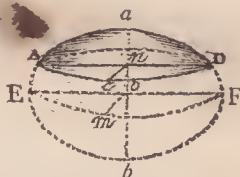
RULE.

1. Divide the fixed axe by the revolving axe, and multiply the quotient by the difference between three times the revolving axe and twice the height of the segment.

2. Multiply the product, thus found, by the square of the height of the segment, and this product again by .5236, and it will give the solidity required.

EXAMPLES.

1. In the prolate spheroid $aEbF$, the transverse axe EF is 100, the conjugate ab 60, and the height an , of the segment aAD , 12: what is the solidity?



Here 156 ($=$ dif. of $3ab$ and $2an$) $\times \frac{12}{3}$ ($= EF$, or $\frac{100}{ab}$) $\times 144$ ($=$ square of an) $\times .5236 = \frac{156 \times 5}{3} \times 144$

$$\times .5236 = 52 \times 5 \times 144 \times .5236 = 260 \times 144 \times .5236 = 37440 \times .5236 = 19603.584 = \text{solidity required.}$$

2. Required the content of the segment of a prolate spheroid; its height being 6, and the axes 40 and 24.

Ans. 2450.44226.

PROBLEM XXIII.

To find the solidity of an elliptic spindle.

RULE.

1. From three times the square of the middle diameter take 4 times the square of the diameter between the middle and the end; and from 4 times this last diameter take 3 times the said middle diameter; and $\frac{1}{4}$ of the quotient arising from dividing the former difference by the latter will give the *central distance*.

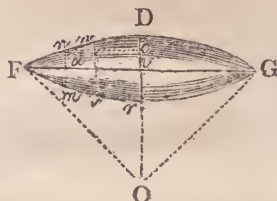
2. Find the axes of the ellipsis by problem the 2d, in conic sections, and the area of the generating segment by problem the 5th.

3. Divide 3 times the area, thus found, by the length of the spindle, and from the quotient subtract the middle diameter; then multiply the remainder by 4 times the central distance, and subtract the product from the square of the middle diameter; and this difference multiplied by $\frac{1}{3}$ of the length of the spindle, and the product again by 1.57079, will give the solidity.

EXAMPLES.

1. What is the solidity of the elliptic spindle Fr GD, whose length FG is 80, the middle diameter

Dr 24, and the diameter *ws* at $\frac{1}{4}$ of the length 18.99094?



Here $(3 \times 24^2 - 18.99094^2 \times 4 \div 18.99094 \times 4 - 3 \times 4) \times \frac{1}{4} = (1728 - 1442.62520833 \div 75.96376 - 72) \times \frac{1}{4} = (285.37679167 \div 3.96376) \times \frac{1}{4} = 71.996 \times \frac{1}{4} = 17.999 = 18$ nearly, for the central distance Ov.

And $18 + \frac{24}{2} = 18 + 12 = 30$ conjugate DO. And $\frac{30}{2} \times 30 \div \sqrt{50^2 - 18^2} = 40 \times 30 \div \sqrt{900 - 324} = 1200 \div \sqrt{576} = \frac{1200}{24} = 50 = \frac{1}{2}$ transverse.

Also $\frac{12}{2 \times 30} = \frac{12}{60} = \frac{1}{5} = .2 = \text{tab. versed sine}$; the circular segment to which is .111823 $\therefore .111823 \times 100$ (AB) $\times 60$ (DF) $= 11.1823 \times 60 = 670.938 =$ generating segment FDG.

Whence $24^2 - [(\frac{670.938 \times 3}{80} - 24) \times 72 \text{ or } 40v] = 24^2 - (25.160175 - 24 \times 72) = 24^2 - 1.160175 \times 72 = 24^2 - 83.5326 = 576 - 83.5326 = 492.4674$. And $492.4674 \times \frac{80}{3} \times 1.57079 = 13132.464 \times 1.57079 = 20628.34312656 = \text{solidity required}$.

2. The length of an elliptic spindle is 40, the middle diameter 12, and the diameter at $\frac{1}{4}$ of the length 9.49546: what is the solidity?

Ans. 2578.56.

PROBLEM XXIV.

To find the solidity of the middle frustum of an elliptic spindle; the length, the diameters of the middle and end, and another parallel thereto at $\frac{1}{4}$ of the length of the frustum, being given.

RULE.

1. From the sum of 3 times the square of the middle diameter and the square of that of the end take 4 times the square of the diameter between the middle and the end; and from 4 times the last diameter take the sum of the least diameter and 3 times that of the middle, and $\frac{1}{4}$ of the quotient arising from dividing the former difference by the latter will give the *central distance*.

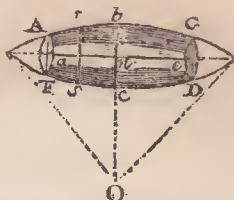
2. Find the axes of the ellipse by problem the 2d, and the area of the elliptical segment whose chord is the length of the frustum by problem the 5th.

3. Divide three times the area thus found, by the length of the frustum, and from the quotient subtract the difference between the middle diameter and that of the end, and multiply the remainder by 8 times the central distance.

4. Then from the sum of the square of the least diameter, and twice the square of that in the middle, take the product last found, and this difference multiplied by the length, and the product again by .261799, &c. will give the solidity required.

EXAMPLES.

1. What is the solidity of the frustum AFDG, whose length ae is 28, the middle diameter bc 24, the diameter AF of the end 21.6, and that rs at $\frac{1}{4}$ of the length 23.469?



$$\text{Here } \frac{24^2 \times 3 + (21.6)^2 - 4 \times (23.409)^2}{4 \times 23.409 - (21.6 + 3 \times 24)} = \frac{2194.56}{93.63636}$$

$$\frac{2191.925124}{-93.6} = \frac{2194.56 - 2191.925124}{.03636} = \frac{2.634876}{.03636}$$

$$= 72.46.$$

And $\frac{72.46}{4} = 18.11 = 18 \text{ nearly} = \text{central distance on.}$

Also $\frac{30 (=bo) \times 14 (=an)}{\sqrt{30^2 (=bo^2) - 28.8^2 (=om^2)}} = \frac{420}{\sqrt{900 - 829.44}}$

$$= \frac{420}{\sqrt{70.56}} = \frac{420}{8.4} = 50 = \frac{1}{2} \text{ transverse diameter.}$$

And $\frac{1.2 (=bm)}{60 (=bp)} = .02 = \text{tab. versed sine; and}$

$.003748 = \text{circular segment belonging to } .02; \therefore$

$.003748 \times 100 (=LN) \times 60 (=bp) = 22.488 =$

$\text{elliptic segment, of which } AbG \text{ is the arc.}$

Whence $24^2 \times 2 + (21.6)^2 - \left(\frac{3 \times 22.488}{28} - 2.4 (= \text{diff. of } bc \text{ and } AF) \right) \times 8 \times 18 = 1152 + 466.56 - (2.4094 - 2.4 \times 144) = 1618.56 - .0094 \times 144 = 1618.56 - 1.3536 = 1617.2064.$

And $1617.2064 \times 28 (=ac) \times .261799 = 45281.7792$

$\times .261799 = 11854.7245127808 = \text{solidity required.}$

2. In the middle frustum of an elliptic spindle, the middle diameter is 32, the diameter at the end 24, and the diameter at $\frac{1}{4}$ of the length 30.15756, and the length 40: required the solidity. *Ans.* 27419.8219.

PROBLEM XXV.

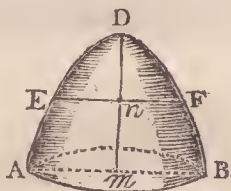
To find the solidity of a parabolic concoid.

RULE.

Multiply the area of the base by half the altitude, and the product will be the content.

EXAMPLES.

1. What is the solidity of the paraboloid ADB, whose height Dm is 84, and the diameter BA of its circular base 48?



Here $48^2 \times .7854 \times 42 (= \frac{1}{2} Dm) = 2304 \times .7854 \times 42$
 $= 1809.5616 \times 42 = 76001.5872 \doteq \text{solidity required.}$

2. What is the solidity of a paraboloid, whose height is 60, and the diameter of its circular base 100? *Ans.* 235620.

3. Required the solidity of a parabolic conoid, whose height is 30, and the diameter of its base 40?

Ans. 18849.6.

4. Required the solidity of a parabolic conoid, whose height is 50, and the diameter of its base 100?

Ans. 196350.

PROBLEM XXVI.

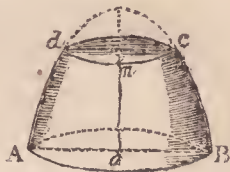
To find the solidity of the frustum of a paraboloid, when its ends are perpendicular to the axe of the solid.

RULE.

Multiply the sum of the squares of the diameters of the two ends by the height of the frustum, and the product again by .3927, and it will give the solidity.

EXAMPLES.

1. Required the solidity of the parabolic frustum $ABcd$, the diameter AB of the greater end being 58, that of the lesser end dc 30, and the height ne 18.



Here $(58^2 + 30^2) \times 18 \times .3927 = (3364 + 900) \times 18 \times .3927 = 4264 \times 18 \times .3927 = 76752 \times .3927 = 30140.5104 = \text{solidity required.}$

2. What is the solidity of the frustum of a parabolic conoid, the diameter of the greater end being 60, that of the lesser end 48, and the distance of the ends 18? *Ans.* 41733.0144.

PROBLEM XXVII.

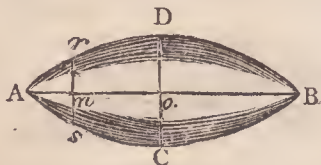
To find the solidity of a parabolic spindle.

RULE.

Multiply the square of the middle diameter by the length of the spindle, and the product again by .418879, and it will give the solidity.

EXAMPLES.

1. The length of the parabolic spindle ACBD is 60, and the middle diameter DC 34: what is the solidity?



Here $34^2 \times 60 \times .418879 = 1156 \times 60 \times .418879 = 69360 \times .418879 = 29053.44744 = \text{solidity required.}$

2. The length of a parabolic spindle is 9 feet, and the middle diameter 3 feet: what is the solidity?

Ans. 33.929192.

PROBLEM XXVIII.

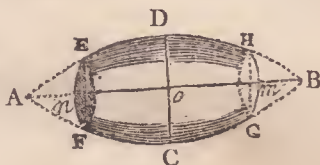
To find the solidity of the middle frustum of a parabolic spindle.

RULE.

Add 8 times the square of the middle diameter, 3 times the square of the less, and 4 times the product of those diameters into one sum; then this sum being multiplied by the length, and the product again by .05236, will give the solidity.

EXAMPLES.

1. In the middle frustum EFGH of the parabolic spindle ACBD, the middle diameter DC is 36, the diameter of the end EF is 20, and the length nm 36: what is the solidity?



Here $(36^2 \times 8 + 20^2 \times 3 + 4 \times 36 \times 20) \times 36 \times .05236$
 $= (10368 + 1200 + 2880) \times 36 \times .05236 = 14448 \times$
 $36 \times .05236 = 520128 \times .05236 = 27233.90228 = \text{solidity required.}$

2. Required the solidity of the middle frustum of a parabolic spindle, the middle diameter being 32, the diameter at the end 24, and the length 40.

Ans. 27210.448.

PROBLEM XXIX.

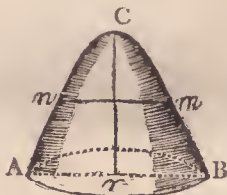
To find the solidity of an hyperboloid.

RULE.

To the square of the radius of the base add the square of the middle diameter between the base and the vertex; and this sum multiplied by the altitude, and the product again by .5236, will give the solidity.

EXAMPLES.

1. In the hyperboloid ACB, the altitude Cr is 10, the radius Ar of the base 12, and the middle diameter nm 15.8745: what is the solidity?



Here $15.8745^2 + 12^2 \times 10 \times .5236 = 251.99975 + 144 \times 10 \times .5236 = 395.99975 \times 10 \times .5236 = 3959.9975 \times .5236 = 2073.4558691 = \text{solidity required.}$

2. In an hyperboloid the altitude is 50, the radius of the base 52, and the middle diameter 68: what is the solidity?

Ans. 191847.

PROBLEM XXX.

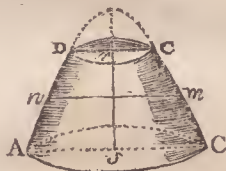
To find the solidity of the frustum of an hyperbolic conoid.

RULE*.

Add together the squares of the greatest and least semi-diameters, and the square of the whole diameter in the middle, then this sum being multiplied by the altitude, and the product again by .52359, will give the solidity.

EXAMPLES.

1. In the hyperbolic frustum ADCB, the length rs is 20, the diameter AB of the greater end 32, that DC of the lesser end 24, and the middle diameter nm 28.1708: required the solidity.



* *Note.* The content of any spindle formed by the revolution of a conic section about its axis may be found by the following rule:

Add together the squares of the greatest and least diameters, and the square of double the diameter in the middle between the two, and this sum multiplied by the length, and the product again by .1309, will give the solidity.

And the rule will never deviate much from the truth when the figure revolves about any other line which is not the axis.

Here $(16^2 + 12^2 + 28.1708^2) \times 20 \times .52359 = (256 + 144 + 793.5939) \times 20 \times .5239 = 1193.5939 \times 20 \times .5239 = 23871.878 \times .5239 = 12499.07660202 = \text{solidity required.}$

2. What is the solidity of the frustum of any hyperbolic conoid, whose greater diameter is 96, lesser diameter 54, the middle diameter 76.4264392, and the altitude 25? *Ans.* 116160.66.

3. Required the solidity of the frustum of an hyperbolic conoid, the height being 12, the greatest diameter 10, the least diameter 6, and the middle diameter $8\frac{1}{2}$? *Ans.* 667.59.

4. What is the content of the middle frustum of an hyperbolic spindle, the length being 20, the middle or greatest diameter 16, the diameter at each end 12, and the diameter at $\frac{1}{4}$ of the length $14\frac{1}{2}$? *Ans.* 3248.969.

5. Required the content of the segment of any spindle, its length being 10, the greatest diameter 8, and the middle diameter 6. *Ans.* 272.272

OF THE REGULAR BODIES.

A *REGULAR BODY* is a solid contained under a certain number of similar and equal plane figures.

The whole number of regular bodies which can possibly be formed is five.

1. The *Tetraedron*, or regular pyramid, which has four triangular faces.

2. The *Hexaedron*, or cube, which has six square faces.

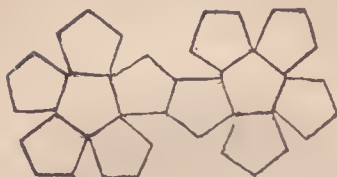
3. The *Octaedron*, which has eight triangular faces.

4. The *Dodecaedron*, which has twelve pentagonal faces.

5. The *Icosaedron*, which has twenty triangular faces.

If the following figures are made of pasteboard, and the lines be cut half through, so that the parts may be turned up and glued together, they will represent the five regular bodies here mentioned.





PROBLEM I.

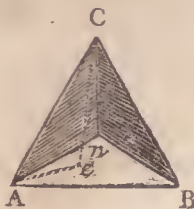
To find the solidity of a tetraedron.

RULE.

Multiply $\frac{1}{12}$ of the cube of the linear side by the square root of 2, and the product will be the solidity.

EXAMPLES.

1. The linear side of the tetraedron ABC is 4: what is the solidity?



$$\frac{4^3}{12} \times \sqrt{2} = \frac{4 \times 4 \times 4}{12} \times \sqrt{2} = \frac{4 \times 4}{3} \times \sqrt{2} = \frac{16}{3} \sqrt{2}$$

$$= \frac{16}{3} \times 1.414 = \frac{22.624}{3} = 7.5413 = \text{solidity required.}$$

2. Required the solidity of a tetraedron whose side is 6?
Ans. 25.452.

PROBLEM II.

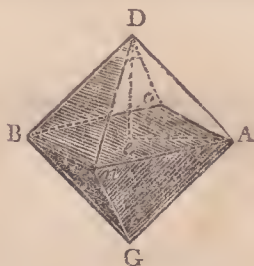
To find the solidity of an octaedron.

RULE.

Multiply $\frac{1}{3}$ of the cube of the linear side by the square root of 2, and the product will be the solidity.

EXAMPLES.

1. What is the solidity of the octaedron BGAD, whose linear side is 4?



$$\frac{4^3}{3} \times \sqrt{2} = \frac{64}{3} \times \sqrt{2} = 21.333, \&c. \times \sqrt{2} = 21.333,$$

$$\&c. \times 1.414, \&c. = 30.16486 = \text{solidity required.}$$

2. Required the solidity of an octaedron, whose side is 8?
Ans. 241.3568.

PROBLEM III.

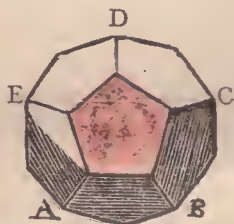
To find the solidity of a dodecaedron.

RULE.

To 21 times the square root of 5 add 47, and divide the sum by 40: then the square root of the quotient being multiplied by 5 times the cube of the linear side will give the solidity required.

EXAMPLES.

1. The linear side of the dodecaedron ABCDE is 3: what is the solidity?



$$\sqrt{\frac{21\sqrt{5}+47}{40}} \times 27 \times 5 = \sqrt{\frac{21 \times 2.23606 + 47}{40}} \times 27 \times 5 = \sqrt{\frac{46.95726 + 47}{40}} \times 135 = 206.901, \text{ solidity required.}$$

2. The linear side of a dodecaedron is 1: what is the solidity?

Ans. 7.6631.

PROBLEM IV.

To find the solidity of an icosaedron.

RULE*.

To 3 times the square root of 5 add 7, and divide the sum by 2; then the square root of this quotient being multiplied by $\frac{5}{6}$ of the cube of the linear side will give the solidity required.

That is, $\frac{5}{6} S^3 \times \sqrt{\left(\frac{7+3\sqrt{5}}{2}\right)} = \text{solidity when } S =$
to the linear side.

EXAMPLES.

1. The linear side of the icosaedron ABCDEF is 3: what is the solidity?

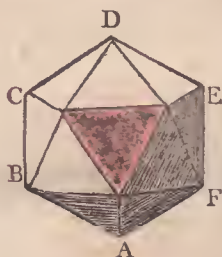
* *Note.* The superficies and solidity of any of the 5 regular bodies may be found as follows.

RULE. 1. Multiply the tabular area by the square of the linear edge, and the product will be the superficies.

2. Multiply the tabular solidity by the cube of the linear edge, and the product will be the solidity.

Surfaces and Solidities of the Regular Bodies.

No. of sides.	Names.	Surfaces.	Solidities.
4	Tetraedron	1.73205	0.11785
6	Hexaedron	6.00000	1.00000
8	Octaedron	3.46410	0.47140
12	Dodecaedron	20.64573	7.66312
20	Icosaedron	8.66025	2.18169



$$\begin{aligned} \sqrt{\frac{3\sqrt{5}+7}{2}} \times \frac{5 \times 3^3}{6} &= \sqrt{\frac{3 \times 2.23606 + 7}{2}} \times \frac{5 \times 27}{6} = \\ \sqrt{\frac{6.70818 + 7}{2}} \times \frac{5 \times 9}{2} &= \sqrt{\frac{13.70818}{2}} \times \frac{45}{2} = 2.61803 \times \\ 22.5 &= 58.90422 = \text{solidity required.} \end{aligned}$$

2. Required the solidity of an icosaedron, whose linear side is 1. Ans. 2.1816949905.

OF

CYLINDRIC RINGS.

PROBLEM I.

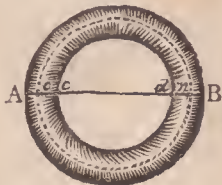
To find the convex superficies of a cylindric ring.

RULE.

TO the thickness of the ring add the inner diameter, and this sum being multiplied by the thickness, and the product again by 9.8696, will give the superficies required.

EXAMPLES.

1. The thickness Ac of a cylindric ring is 3 inches, and the inner diameter cd 12 inches: what is the convex superficies?



$$\overline{12+3} \times 3 \times 9.8696 = 15 \times 3 \times 9.8696 = 45 \times 9.8696 = 444.132 = \text{superficies required.}$$

2. The thickness of a cylindric ring is 4 inches, and the inner diameter 18: what is the convex superficies? *Ans. 868.52 square inches.*

3. The thickness of a cylindric ring is 2 inches, and the inner diameter 18: what is the convex superficies? *Ans. 394.785 square inches.*

PROBLEM II.

To find the solidity of a cylindric ring.

RULE*.

To the thickness of the ring add the inner diameter, and this sum being multiplied by the square of half the thickness, and the product again by 9.8696, will give the solidity.

EXAMPLES.

1. What is the solidity of an anchor ring, whose inner diameter is 8 inches, and thickness in metal 3 inches?

$$\overline{8+3 \times \frac{3}{2}}^2 \times 9.8696 = 11 \times 1.5^2 \times 9.8696 = 11 \times 2.25 \times 9.8696 = 24.75 \times 9.8696 = 244.2726 = \text{solidity required.}$$

* This figure being only a cylinder bent round into a ring, its surface and solidity may also be found as in the cylinder, namely, by multiplying the axis or length of the cylinder by the circumference of the ring, or section, for the surface, and by the area of a section for the solidity.

2. The inner diameter of a cylindric ring is 18 inches, and its thickness 4 inches: what is the solidity?

Ans. 868.5248.

3. Required the solidity of a cylindric ring, whose thickness is 2 inches, and its inner diameter 12.

Ans. 138.1744.

4. What is the solidity of a cylindric ring, whose thickness is 4 inches, and inner diameter 26?

Ans. 1184.352.

OF

ARTIFICER'S WORK.

ARTIFICERS estimate or compute the value of their works by different measures, viz.*

1. Mason's flat work, &c. by the foot.
2. Painting, plastering, paving, &c. by the yard.
3. Flooring, partitioning, roofing, tiling, &c. by the square of 100 feet.
4. Brickwork is done in America by the thousand. In some other countries it is estimated by the rod of $16\frac{1}{2}$ feet, whose square is $272\frac{1}{4}$.

The measures made use of in these works are contained in the following table:

12 inches	} make	1 lineal foot
144 square inches		1 square foot
9 square feet		1 square yard
100 square feet		a square
272 $\frac{1}{4}$ square feet, or } 30 $\frac{1}{4}$ square yards }		1 rod, perch, or square pole.

* The best method of taking the dimensions of all sorts of artificer's work is by feet, tenths, and hundredths; because the computations may then be performed by common multiplication, or by the sliding rule, hereafter described.

OF

BRICKLAYER'S WORK.

BRICKLAYERS compute their work by the thousand. But when their work is to be valued, and the number of bricks to be ascertained by measurement, the mode in America is to allow, for a wall nine inches thick, $13\frac{1}{2}$ bricks to every square foot the wall contains; and $19\frac{1}{2}$ bricks for a wall of 14 inches thick. In England and some other countries, when the artificer finds his own materials, they compute or value their work at the rate of a brick and a half thick; and if a wall be more or less than this standard, it must be reduced to it, as follows:

RULE.

Multiply the superficial content of the wall in feet, by the number of half bricks in the thickness, and $\frac{1}{3}$ of that product will be the content required.

EXAMPLES.

1. How many square rods are there in a wall $52\frac{1}{2}$ feet long, 12 feet 9 inches high, and $2\frac{1}{2}$ bricks thick*?

* *Note.* In practice it is usual to divide the square feet by 272 only, omitting the $\frac{1}{4}$.

*By Decimals.*52.5 *length*12.75 = *height*

 2625
 3675
 1050
 525

272)669.375(2.4609

544 5 *half bricks*

 1253.3)12.3045

1088 *ro. fc. in.*

 4.1015 = 4 27 7 *the answer.*

1657

1632

 2550

2448

 102

All windows, doors, &c. are to be deducted out of the contents of the walls in which they are placed.

All ornamental work is generally valued by the foot square, such as arches, doors, architraves, frizes, cornices, &c. But carved mouldings, &c. are often agreed for by the running foot, or lineal measure.

By Cross Multiplication.

$$\begin{array}{r}
 \text{feet in.} \\
 52 \quad 6 \\
 12 \quad 9 \\
 \hline
 630 \quad 0 \\
 39 \quad 4 \quad 6 \\
 \hline
 272 \overline{)669 \quad 4 \quad 6} \\
 \hline
 2 \quad 124 \quad 4 \\
 \quad 5 \\
 \hline
 3 \overline{)12 \quad 82 \quad 8} \\
 \hline
 4 \quad 27 \quad 7 \text{ as before.}
 \end{array}$$

2. How many square rods are there in a wall $62\frac{1}{2}$ feet long, 14 feet 8 inches high, and $2\frac{1}{2}$ bricks thick?
ro. fe. in. p.

Ans. 5 167 9 4

3. If each side wall of a building be 45 feet long on the outside, each end wall 15 feet broad on the inside, the height of the building 20 feet, and the gable at each end of the wall 6 feet high, the whole being 2 bricks thick: what is the true content in standard rods?
Ans. 12.1761.

OF

MASON'S WORK.

TO Masonry belongs all sorts of stone work, and the measure made use of is a foot, either superficial or solid.

Walls, blocks of marble or stone, columns, &c. are measured by the solid foot; and pavements, slabs, chimney-pieces, &c. by the superficial foot*.

EXAMPLES.

1. Required the solid content of a wall whose length is 48 feet 6 inches, its height 10 feet 9 inches, and thickness 2 feet.

* Solid measure is principally used for materials, and the superficial for workmanship.—In the solid measure the true length, breadth, and thickness are taken, and multiplied continually together. And in the superficial measure, the length and breadth of every part of the projection must be taken, as it appears without the general upright face of the building.

All windows, doors, &c. are to be deducted out of the contents of the walls in which they are placed; but this deduction is only to be made with regard to materials, for the value of the workmanship is to be added to the workman's bill at the stated rate only.

By Decimals.

$$\begin{array}{r}
 48.5 \\
 10.75 \\
 \hline
 2425 \\
 3395 \\
 4850 \\
 \hline
 521\ 375 \\
 2 \\
 \hline
 1042.750 \text{ the answer.}
 \end{array}$$

By Cross Multiplication.

$$\begin{array}{r}
 \text{feet in.} \\
 48\ 6 \\
 10\ 9 \\
 \hline
 485\ 0 \\
 36\ 4\ 6 \\
 \hline
 521\ 4\ 6 \\
 2\ 0\ 0 \\
 \hline
 1042\ 9\ 0 \text{ the same as before.}
 \end{array}$$

2. Required the solid content of a wall whose length is 53 feet 6 inches, its height 12 feet 3 inches, and its thickness 2 feet. *Ans. 1310 feet 9 in.*

3. What is a marble slab worth, whose length is 5 feet 7 inches, and breadth 1 foot 10 inches, at 80 cts. per foot? *Ans. 8 18 8*



4. What is the solid content of a wall, whose length is 60 feet 9 inches, its height 10 feet 3 inches, and its thickness $2\frac{1}{2}$ feet? *Ans. 1556.71875 feet.*

5. In a chimney-piece, suppose the		<i>fe. in.</i>	
Length of the mantle and slab each		4	6 }
Breadth of both together	-	3	2 }
		<i>fe. in.</i>	
Length of each jamb	-	4	4 }
Breadth of both together	-	1	9 }
What will be the content of the chimney-piece?			
<i>Ans. 21 feet 10 in.</i>			

OF

CARPENTER'S WORK.

CARPENTER'S work is that of flooring, partitioning, roofing, &c. and is measured by the square of 100 feet.

EXAMPLES.

1. If a floor be 57 feet 3 inches long, and 28 feet 6 inches broad: how many squares will it contain?

By Decimals.

$$\begin{array}{r}
 57.25 \\
 28.5 \\
 \hline
 28625 \\
 45800 \\
 11450 \\
 \hline
 16|31.625
 \end{array}$$

By Cross Multiplication.

$$\begin{array}{r}
 \text{feet in.} \\
 57 \quad 3 \\
 28 \quad 6 \\
 \hline
 1603 \quad 0 \\
 28 \quad 7 \quad 6 \\
 \hline
 16|31 \quad 7 \quad 6.
 \end{array}$$

2. A floor is 53 feet 6 inches long, and 47 feet 9 inches broad: how many squares will it contain?

Ans. 25 sq. and 54 feet.

3. A partition is 91 feet 9 inches long, and 11 feet 3 inches broad: how many squares will it contain?

Ans. 10 sq. and 32 feet.



4. If a house within the walls be 44 feet 6 inches long, and 18 feet 3 inches broad: how many squares of roofing will cover it?

Ans. 12 sq. and 18 feet.

5. If a house measure within the walls 52 feet 8 inches in length, and 30 feet 6 inches in breadth, and the roof be of a true pitch, what will it cost roofing at 1. *doll.* 40 *cts.* per square?

dolls. cts. m.

Ans. 33 71 9

OF

SLATER'S AND TILER'S WORK.

IN these works, the content of a roof is found by multiplying the length of the ridge by the girt from eave to eave; and, in slating, allowance must be made for the double row at the bottom.

In taking the girt, the line is made to ply over the lowest row of slates, and returned up the under-side till it meet with the wall or eaves-board; but in tiling, the line is stretched down only to the lowest part, without returning it up again.

Double measure is generally allowed for hips, vallies, gutters, &c. but no deductions are made for chimnies.

EXAMPLES.

1. The length of a slated roof is 45 feet 9 inches, and its girt 34 feet 3 inches: what is the content?

By Decimals.

$$\begin{array}{r}
 45.75 \\
 34.25 \\
 \hline
 22875 \\
 9150 \\
 18300 \\
 13725 \\
 \hline
 9)1566.9375 \\
 \hline
 174.104
 \end{array}$$

SLATER'S AND TILER'S WORK. 189

By Cross Multiplication.

feet. in.

45 9
34 3

1555 6
11 5 3

9)1566 11 3

174 0 11 3

Ans. 174 yards.

2. What will the tiling a barn cost at 3 dolls. 40 cts. per square, the length being 43 feet 10 inches, and the breadth 27 feet 5 inches, on the flat, the eave-boards projecting 16 inches on each side?

Ans. 65 dolls. 17 cts.



PLASTERER'S WORK.

PLASTERER'S work is of two kinds, viz. plastering upon laths, called cieling ; and plastering upon walls ; and these different kinds must be measured separately, and the contents of each valued by the price agreed on.

Note. Proper deductions must be made for doors, windows, &c.

EXAMPLES.

If a cieling be 59 feet 9 inches long, and 24 feet, 6 inches broad : how many yards does it contain ?

By Decimals.

$$\begin{array}{r}
 59.75 \\
 24.5 \\
 \hline
 29875 \\
 23900 \\
 11950 \\
 \hline
 9)1463.875 \\
 \hline
 162.652
 \end{array}$$

Plasterer's plain work is measured by the square foot, or yard of 9 square feet, and enriched mouldings, &c. by running or lineal measure.

By Cross Multiplication.

	feet	in.
	59	9
	24	6
<hr/>		
	1434	0
	29	10 6
<hr/>		
9)	1453	10 6
<hr/>		
	162	5 10 6

Ans. 162 yards 5 feet.

2. If the partitions between rooms be 141 feet 6 inches about, and 11 feet 3 inches high : how many yards do they contain ?

Ans. 176.87.

3. There is a quantity of partitioning that measures 234 feet 8 inches about, and 14 feet 6 inches high, and is rendered between quarters ; the lathing and plastering of which will be 8 *cts.* per yard, and the whiting 2 *cts.* per yard: what will the whole come to ?

Ans. 33 dolls. 26 cts.

4. The length of a room is 14 feet 5 inches, its breadth 13 feet 2 inches, and height 9 feet 3 inches to the under side of the cornice, whose girt is $8\frac{1}{2}$ inches, and its projection 5 inches from the wall on the upper part next the cieling : what will be the quantity of plastering, supposing there are no deductions but for one door, whose size is 7 feet by 4 feet ?

Ans. 53 yards 5 feet 3 inches, of rendering, 18 yards 5 feet 6 inches, of cieling, and 39 feet $0\frac{1}{11}$ inches, of cornice.

The work is chiefly of two kinds ; viz. plastering upon laths, called cieling ; and plastering upon walls, all the different kinds are to be measured separately ; and then the contents of the same kind, or the same number of coats must be collected together into one sum, and that sum multiplied by the stipulated price per yard.

OF

PAINTER'S & GLAZIER'S WORK.

PAINTER'S work is measured in the same manner as carpenter's work, and is estimated at a given price per yard, except the painting of sashes, which is charged at so much per light.

Glazier's work is done by the light, that is, computed at a given price for putting in each pane of glass, according to the size.

EXAMPLES.

1. If a pane of glass be 2 feet 8 inches and 3 quarters long, and 1 foot 4 inches 1 quarter broad: how many feet does it contain?

By Decimals.

The decimal of $8\frac{3}{4}$ inches is .729

And that of $4\frac{1}{2}$ inches is .354

2.729

1.354

10916

13645

8187

2729

3.695066

12

8.340792

PAINTER'S WORK.

193

2. If a room be painted, whose height is 16 feet 6 inches, and its compass 97 feet 9 inches: how many yards does it contain?

By Decimals.

$$\begin{array}{r}
 97.75 \\
 16.5 \\
 \hline
 48875 \\
 58650 \\
 9775 \\
 \hline
 9)1612.875 \\
 \hline
 179.208
 \end{array}$$

By Cross Multiplication.

$$\begin{array}{r}
 \text{feet} \quad \text{in.} \\
 97 \quad 9 \\
 16 \quad 6 \\
 \hline
 1564 \quad 0 \\
 48 \quad 10 \quad 6 \\
 \hline
 9)1612 \quad 10 \quad 6 \\
 \hline
 179 \quad 1 \quad 10 \quad 6
 \end{array}$$

Ans. 179 yards 1 foot.

2. The height of a room is 14 feet 10 inches, and the circumference 21 feet 8 inches: how many square yards does it contain?

Ans. 35.

3. Suppose a room, that was to be painted at 8*d.* per yard, measures as follows: The height is 11 *fe.* 7 *in.* the girt or compass 74 *fe.* 10 *in.* the door 7 *fe.* 6 *in.* by 3 *fe.* 9 *inches*; five window shutters, each 6 *fe.* 8 *in.* by 3 *fe.* 4 *in.* the breaks in the windows 14 *in.* deep, and 8 *fe.* high; the chimney 6 *fe.* 9 *in.* by

R

5 *fe.*, a closet, the height of the room, $3\frac{1}{2}$ *fe.* deep, and $4\frac{3}{4}$ *fe.* in front, with shelving, at 22 *fe.* 6 *in.* by 10 *in.*, the shutters, doors, and shelves, being all coloured on both sides: what will the whole come to?

Ans. 4*l.* 18*s.* 9*d.*



Note. Painters take their dimensions with a string, and measure from the top of the cornice to the floor, girting the string over all the mouldings and swellings; and their price is generally proportioned to the number of times they lay on their colour.

All work of this kind is done by the square yard, and every part where the colour lies must be measured, and estimated in the general account of the work.

Deductions are to be made for chimnies, casements, &c. and the price is generally proportioned to the number of times they lay on their colour.

Painters generally measure their work to a quarter of an inch, and all circular, triangular, &c. windows, are measured as if they were squares.

OF

PAVIOUR'S WORK*.

PAVIOUR'S work is done by the square yard, and the content is found by multiplying the length by the breadth.

Or, if the dimensions be taken in feet, and the area be found in the same measure, the result being divided by 9, will give the number of square yards required.

EXAMPLES.

1. What will the paying a rectangular court-yard come to at 3s. 2d. per yard, supposing the length to be 27 feet 10 inches, and the breadth 14 feet 9 inches?

By Cross Multiplication.

	<i>feet</i>	<i>in.</i>	
	27	10	
	14	9	
	<hr/>		
	389	8	
	20	10 6	
	<hr/>		
9)	410	6 6	
	<hr/>		
	45	5 6 6	

* Plumber's work is generally done by the pound, or hundred weight, and the price is regulated according to the value of the lead at the time the contract is made, or when the work is performed.

		45		5	6	6 at 3s. 2d.		
s.	d.							
2	0	$\frac{1}{10}$	45					
1	0	$\frac{1}{2}$	4	10				
	2	$\frac{1}{6}$	2	5				
f.	in.	p.						
3	0	0	$\frac{1}{3}$	7	6			
1	0	0	$\frac{1}{3}$	1	$0\frac{1}{2}$			
1	0	0	$\frac{1}{3}$	0	4			
0	6	0	$\frac{1}{2}$	0	4			
0	0	6	$\frac{1}{12}$	0	2			
				0	0			
			7 4 $4\frac{1}{2}$ the answer.					

2. A rectangular court-yard is 42 feet 9 inches long, and 68 feet 6 inches in depth, and a foot-way goes quite through it, of five feet 6 inches in breadth; the foot-way is laid with stone at 3s. 6d. per yard, and the rest with pebbles at 3s. per yard: what will the whole come to?

Ans. 49l. 17s. $0\frac{1}{4}$ d.

Sheet-lead used in roofing, guttering, &c. is generally between 7 and 12lbs. weight to the square foot.

The following table will shew the weight of a square foot to each of these thicknesses.

Thick.	lbs. sq. foot.	Thick.	lbs. sq. foot.	Thick.	lbs. sq. foot.
$\frac{1}{8}$	7.373	.15	8.848	.18	10.618
.13	7.668	.16	9.438	.19	11.207
.14	8.258	$\frac{1}{6}$	9.831	$\frac{1}{3}$	11.797
$\frac{1}{7}$	8.427	.17	10.028	.21	12.387

OF

VAULTED AND ARCHED ROOFS.

ARCHED roofs are either *vaults*, *domes*, *saloons*, or *groins*.

Vaulted roofs are formed by arches springing from the opposite walls, and meeting in a line at the top.

Domes are made by arches springing from a circular or polygonal base, and meeting in a point at the top.

Saloons are formed by arches connecting the side walls to a flat roof, or cieling in the middle.

Groins are formed by the intersection of vaults with each other.

Domes and Saloons rarely occur in the practice of measuring, but vaults and groins cover the cellars of most houses.

Vaulted roofs are generally one of the three following sorts :

1. *Circular roofs*, or those whose arch is some part of the circumference of a circle.
2. *Elliptical roofs*, or those whose arch is some part of the circumference of an ellipsis.
3. *Gothic roofs*, or those which are formed by two circular arcs that meet in a point directly over the middle of the breadth, or span of the arch.

PROBLEM I.

To find the solid content of circular, elliptic, or gothic vaulted roofs.

RULE*.

Multiply the area of one end by the length of the roof, and the product will be the solidity required.

EXAMPLES.

1. What is the solid content of a semi-circular vault, whose span is 40 feet, and its length 120 feet?

$$\begin{array}{r} .7854 \\ 1600 = \text{square of } 40. \end{array}$$

$$\begin{array}{r} 4712400 \\ 7854 \end{array}$$

$$\hline 2)1256.6400$$

$$\begin{array}{r} 628.32 = \text{area of the end.} \\ 120 = \text{length.} \end{array}$$

$$\hline 75398.40 = \text{solidity required.}$$

2. Required the solidity of an elliptic vault, whose span is 40 feet, height 12 feet, and length 80.

Ans. 30159.36 feet.

* *To find the solidity of the materials in either of the arches.*

Rule. From the solid content of the whole arch take the solid content of the void space, and the remainder will be the solidity of the arch.

VAULTED AND ARCHED ROOFS. 199

3. What is the solid content of a gothic vault, whose span is 48, the chord of its arch 48, the distance of the arch from the middle of the chord 18, and the length of the vault 60?

Ans. 136228.044.

PROBLEM II.

To find the concave, or convex surface, of circular, elliptic, or gothic vaulted roofs.

RULE*.

Multiply the length of the arch by the length of the vault, and the product will be the superficies required.

EXAMPLES.

1. What is the concave surface of a semi-circular vault, whose span is 40 feet, and its length 120?

$$\begin{array}{r}
 3.1416 \\
 40 \\
 \hline
 2)125.6640 \\
 \hline
 62.832 = \text{length of the arch.} \\
 120 \\
 \hline
 7539.840 = \text{concave surface required.}
 \end{array}$$

* The convex surface of a vault may be found by stretching a string over it; but for the concave surface this method is not applicable, and therefore its length must be found from proper dimensions.

PROBLEM III.

To find the solid content of a dome ; its height, and the dimensions of its base being known.

RULE*.

Multiply the area of the base by $\frac{2}{3}$ of the height, and the product will be the solidity.

EXAMPLES.

1. What is the solid content of a spherical dome, the diameter of whose circular base is 60 feet ?

$$.7854$$

$$3600 = \text{square of } 60.$$

$$\begin{array}{r} 4712400 \\ 23562 \end{array}$$

$$2827.4400 = \text{area of the base.}$$

$$20 = \frac{2}{3} \text{ of the height (30).}$$

$$56548.8000 = \text{solidity required.}$$

2. In an hexagonal spherical dome, one side of the base is 20 feet : what is the solidity ?

Ans. 12000 feet.

PROBLEM IV.

To find the superficial content of a spherical dome.

* Domes and saloons are of various figures, but they are things that seldom occur in the practice of measuring.

RULE*.

Multiply the area of the base by 2, and the product will be the superficial content required.

EXAMPLES.

1. What will the painting an hexagonal spherical dome come to at 1s. per yard; each side of the base being 20 feet?

2.598076 = area of a hexagon whose side is 1.
400 = square of 20.

1039.230400 = area of the base.

$\frac{2}{2}$

9)2078.460800 = superficial content required.

2.0)230.940088

11.5470044 = 11l. 10s. 11d. the expence of [painting.

PROBLEM V.

To find the solid content of a saloon.

* The practical rule for elliptical domes is as follows:

Rule. Add the height to half the diameter of the base, and this sum multiplied by 1.5708 will give the superficial content nearly.

RULE*.

1. Multiply the height of the arc, its projection, $\frac{1}{4}$ of the perimeter of the cieling, and 3.1416 continually together, and call the product A.

2. From a side or diameter of the room take a like side or diameter of the cieling, and multiply the square of the remainder by the proper factor (page 65) and this product again by $\frac{2}{3}$ of the height, and call the last product B.

3. Multiply the area of the flat cieling by the height of the arch, and this product added to the sum of A and B will give the content required.

Note. When the arch is not a quadrant, find the area of a circular semi-segment, whose versed sine and right sine are respectively equal to the projection and height of the arch; multiply this area by the perimeter of the cieling, and call the product A.

* *To find the superficial content of a saloon.*

1. Find the area of the flat part of the cieling.

2. Find the convex surface of a cylinder, or cylindroid, whose length is equal to $\frac{1}{4}$ the perimeter of the cieling, and its diameters to twice the height and twice the projection of the arch.

3. Find the superficial content of a dome of the same figure as the arch, and whose base is either a square, or a figure similar to that of the cieling; the side being equal to the difference of a side of the room, and a side of the cieling.

4. Add these three articles together, and the sum will give the superficial content required.

Note. In a rectangular, circular, or polygonal room, the base of the dome will be a square, a circle, or a like polygon.

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EXAMPLES.

1. What is the solid content of a saloon with a circular quadrantal arch of 2 feet radius, springing over a rectangular room of 20 feet long and 16 feet wide?

Here the flat part of the cieling is 16 feet by 12; and

$$4)56$$

$$14 = \frac{1}{4} \text{ of the perimeter.}$$

$$2 = \text{height.}$$

$$28$$

$$2 = \text{projection.}$$

$$56$$

$$3.1416$$

$$56$$

$$188496$$

$$157080$$

$$175.9296 = A.$$

$$20 = \text{side of the room.}$$

$$16 = \text{side of the cieling.}$$

$$4$$

$$4$$

$$16$$

$$1.000, \&c. = \text{factor.}$$

$$16.000$$

$$1\frac{1}{3} = \frac{2}{3} \text{ of the height.}$$

$$16.000$$

$$5.333$$

$$21.333 = B.$$

204 VAULTED AND ARCHED ROOFS.

16

12

192 = *area of the flat cieling.*

2 = *height of the arch.*

384

175.9296

21.3333

581.2629 = *solid content required.*

2. A circular building of 40 feet diameter, and 25 feet high to the cieling, is covered with a saloon, whose circular quadrantal arch is 5 feet radius: required the capacity of the room in cubic feet.

Ans. 30779.45948 feet.

PROBLEM VI.

To find the solid content of the vacuity formed by a groin arch, either circular or elliptical.

RULE.

Multiply the area of the base by the height, and the product again by .904, and it will give the solidity required.

EXAMPLES.

1. What is the solid content of the vacuity formed by a circular groin, one side of its square base being 12 feet?

$$\begin{array}{r}
 12 \\
 12 \\
 \hline
 144 = \text{area of the base.} \\
 6 = \text{height.} \\
 \hline
 864 \\
 .904 \\
 \hline
 3456 \\
 77760 \\
 \hline
 781.056 = \text{solidity required.}
 \end{array}$$

2. What is the solid content of the vacuity formed by an elliptical groin, one side of its square base being 20 feet, and the height 6 feet?

Ans. 2169.6

PROBLEM VII.

To find the concave superficies of a circular groin.

RULE*.

Multiply the area of the base by 1.1416, and the product will be the superficies required.

* This rule may also be observed in elliptical groins, the error being too small to be regarded in practice.

In measuring works where there are many groins in a range, the cylindric pieces between the groins, and on their sides, must be computed separately.

And to find the solidity of the brick, or stone work, which forms the groin arches, observe the following

S

206 VAULTED AND ARCHED ROOFS.

EXAMPLES.

1. What is the curve superficies of a circular groin arch, one side of its square being 12 feet?

$$\begin{array}{r}
 12 \\
 12 \\
 \hline
 144 = \text{area of the base.} \\
 1.1416 \\
 \hline
 45664 \\
 45664 \\
 11416 \\
 \hline
 164.3904 = \text{superficies required.}
 \end{array}$$

Rule. Multiply the area of the base by the height, including the work over the top of the groin, and this product lessened by the solid content, found as before, will give the solidity required.

The general rule for measuring all arches is this:

From the content of the whole, considered as solid, from the springing of the arch to the outside of it, deduct the vacuity contained between the said springing and the under side of it, and the remainder will be the content of the solid part.

And because the upper sides of all arches, whether vaults or groins, are built up solid, above the haunces, to the same height with the crown, it is evident that the area of the base will be the whole content above-mentioned, taking for its thickness the height from the springing to the top. And for the content of the vacuity to be deducted, take the area of its base, accounting its thickness to be $\frac{2}{3}$ of the greatest inside height. But it may be noted that the area used in the vacuity, is not exactly the same with that used in the solid; for the diameter of the former is twice the thickness of the arch less than that of the latter.

And although I have mentioned the deduction of the vacuity as common to both the vault and the groin, it is rea-

2. What is the concave superficies of a circular groin arch, one side of its square being 9 feet?

Ans. 92.4696.

sonable to make it only in the former, on account of the waste of materials and trouble to the workmen, in cutting and fitting them for the angles and intersections.

Whoever wishes to see this subject more fully handled, may consult *La Théorie et la Pratique de la Géométrie*, par *M. l'Abbé Deidier*; a work in which several parts of Mensuration and Practical Geometry are skilfully handled, the examples being mostly wrought out in an easy familiar manner, and illustrated with observations, and figures very neatly executed.

OF THE CARPENTER'S RULE.

THIS instrument is commonly called *Cogeshall's* sliding rule. It consists of two pieces, of a foot in length each, which are connected together by means of a folding joint.

On one side of the rule, the whole length is divided into inches and half quarters, for the purpose of taking dimensions. And on this face there are also several plane scales, divided by diagonal lines into twelfth parts, which are designed for planning such dimensions as are taken in feet and inches.

On one part of the other face there is a slider, and four lines marked A, B, C, and D; the two middle ones B and C being upon the slider.

Three of these lines A, B, C are double ones, because they proceed from 1 to 10 twice over; and the fourth line D is a single one, proceeding from 4 to 40, and is called the *girt line*.

The use of the double lines A and B is for working proportions, and finding the areas of plane figures. And the use of the girt line D, and the other double line C, is for measuring solids.

When 1 at the beginning of any line is counted 1, then the 1 in the middle will be 10, and the 10 at the end 100. And when 1 at the beginning is counted 10, then the 1 in the middle is 100, and the 10 at the end 1000, &c. and all the small divisions are altered in value accordingly.

Upon the other part of this face there is a table of the value of a load of timber, at all prices, from 6*d.* to 2*s.* a foot.

Some rules have likewise a line of inches, or a foot divided decimally into 10th parts; as well as tables of board measure, timber measure, &c. but these will be best understood from a sight of the instrument.

The Use of the SLIDING RULE.

PROBLEM I.

To find the product of two numbers, as 7 and 26.

RULE.

Set 1 upon A, to one of the numbers (26) upon B: then against the other number (7) on A, will be found the product (182) upon B.

Note. If the third term runs beyond the end of the line, seek it on the other radius, or part of the line, and increase the product 10 times.

PROBLEM II.

To divide one number by another, as 510 by 12.

RULE.

Set the divisor (12) on A, to 1 on B; then against the dividend (510) on A, is the quotient ($42\frac{1}{2}$) on B.

S 2

Note. If the dividend runs beyond the end of the line, diminish it 10 or 100 times, to make it fall on A, and increase the quotient accordingly.

PROBLEM III.

To square any number, as 27.

RULE.

Set 1 upon D to 1 upon C; then against the number (27) upon B will be found the square (729) upon C.

If you would square 270, reckon the one on D to be 100; and then the 1 on C will be 1000, and the product 72900.

PROBLEM IV.

To extract the square root of any number, as 4268.

RULE.

Set 1 upon C, to 1 upon D; then against (4268) the number on C, is (65.3) the root on D.

To value this right you must suppose the 1 on C to be some of these squares 1, 100, 1000, &c. which is the nearest to the given number, and then the root corresponding will be the value of the 1 upon D.

PROBLEM V.

To find a mean proportional between any two numbers, as 27 and 450.

RULE.

Set one of the numbers (27) on C, to the same on D; then against the other number (450) on C, will be the mean (112) on D.

Note. If one of the numbers overruns the line, take the 100th part of it, and augment the answer 10 times.

PROBLEM VI.

Three numbers being given, to find a fourth proportional; suppose 12, 28, and 57.

RULE*.

Set the first number (12) upon A, to the second (28) upon B; then against the third number (57) on A, is the fourth (133) on B.

* The use of the rule in board and timber measure will be shown in what follows :

If the breadth of a board be given; to find how much in length will make a square foot.

Rule. If the board be narrow, it will be found in the table of board measure on the rule; but, if not, shut the rule, and seek the breadth in the line of board measure, running along

Note. If one of the middle numbers runs off the line, take the tenth part of it only, and augment the answer 10 times.

The finding a third proportional is exactly the same, the second number being twice repeated.

Thus, suppose a third proportional was required to 21 and 32.

Set the first 21 on B, to the second 32 on A; then against the second 32 on B, is 48.8 on A, which is the third proportional required.

the rule, from that table; then over against it, on the opposite side, is the length in inches required.

The side of the square of a piece of timber being given; to find how much in length will make a foot solid.

Rule. If the timber be small, it will be found in the table of timber measure on the rule; but, if not, look for the side of the square, in the line of timber measure, running along the rule, from that table, and against it in the line of inches is the length required.

OF

TIMBER MEASURE.

PROBLEM I.

To find the area, or superficial content, of a board or plank.

RULE.

MULTIPLY the length by the breadth, and the product will be the content required.

Note. When the board is tapering, add the breadths of the two ends together, and take $\frac{1}{2}$ the sum for the mean breadth.

By the SLIDING RULE.

Set 12 on B to the breadth in inches on A, then against the length in feet on B, is the content on A, in feet and fractional parts, as required.

EXAMPLES.

1. What is the value of a plank, whose length is 8 feet 6 inches, and breadth 1 foot 3 inches throughout, at $2\frac{1}{2}d.$ per foot?

feet in.

8 6

1 3

8 6

2 1 6

10 7 6 *the content.**2d. is* $\frac{1}{5}$ 1 8 $\frac{1}{2}$ *is* $\frac{1}{4}$ 5*in.*6 *is* $\frac{1}{2}$ $1\frac{1}{4}$ *in.*1 *is* $\frac{1}{8}$ $\frac{1}{4}$

2s. $2\frac{1}{2}d.$ *the answer.**By the SLIDING RULE.*As 12 on B : 15 on A :: $8\frac{1}{2}$ on B : $10\frac{1}{2}$ on A.

2. What is the content of a board, whose length is 5 feet 7 inches, and breadth 1 foot 10 inches?

*feet in. fa.**Ans.* 10 2 103. At $1\frac{1}{2}d.$ per foot, what is the value of a plank, whose length is 12 feet 6 inches, and breadth 11 inches throughout?*Ans.* 1s. 5d.4. Find the value of 5 oaken planks at 3d. per foot, each being $17\frac{1}{2}$ feet long, and their particular breadths as follows, viz. two of $13\frac{1}{2}$ inches in the middle, one of $14\frac{1}{2}$ inches in the middle, and the two remaining ones each 18 inches at the broader end, and $11\frac{1}{4}$ at the narrower.*Ans.* 1l. 5s. $9\frac{1}{4}d.$

PROBLEM II.

To find the solidity of squared or four-sided timber.

RULE*.

Multiply the mean breadth by the mean thickness, and this product again by the length, and it will give the solidity required.

* *Note.* 1. If the stick be equally broad and thick throughout, the breadth and thickness, any where taken, will be the mean breadth and thickness.

2. If the tree tapers regularly from one end to the other, the breadth and thickness, taken in the middle, will be the mean breadth and thickness.

3. If the stick does not taper regularly, but is thicker in some places than in others, let several different dimensions be taken, and their sum divided by the number of them will give the mean dimensions.

This method of finding the mean dimensions is mostly used in practice, but in many cases it is exceedingly erroneous.

The quarter girt, likewise, which is mentioned in the proportion by the sliding rule, is subject to error. It is not the fourth part of the circumference, but the square root of the product arising from multiplying the mean breadth by the mean thickness.

In order to show the fallacy of taking $\frac{1}{4}$ of the girt for the side of a mean square, take the following example :

Suppose a piece of timber to be 24 feet long, and a foot square throughout, and let it be slit into two equal parts, from end to end.

Then the sum of the solidities of the two parts, by the quarter girt method, will be 27 feet, but the true solidity is 24 feet; and if the two pieces were very unequal, the difference would be still greater.

By the SLIDING RULE.

As the length in feet on C : 22 on D :: quarter girt in inches on D : solidity on C.

EXAMPLES.

1. The length of a piece of timber is $20\frac{1}{2}$ feet, the breadth at the greater end is 1 foot 9 inches, and the thickness 1 foot 3 inches; and at the lesser end the breadth is 1 foot 6 inches, and the thickness 1 foot: what is the solidity?

$1.75 = \text{greater breadth}$

$1.5 = \text{lesser breadth.}$

2)3.25

$1.625 = \text{mean breadth.}$

$1.25 = \text{greater thickness.}$

$1.00 = \text{lesser thickness.}$

2)2.25

$1.125 = \text{mean thickness.}$

By Decimals.

1.625

1.125

8125

3250

1625

1625

1.828125

1.828125

20.5

9140625

36562500

 $37.4765625 = \text{content.}$ *By Cross Multiplication.**fe. in. pa.*

1 7 6

1 1 6

1 7 6

1 7 6

9 9

1 9 11 3

20 6

36 6 9 0

10 11 7 6

 $37.5876 = \text{content.}$ *By the SLIDING RULE.*

As 1 upon B : $19\frac{6}{12}$ upon A : : $13\frac{6}{12}$ upon B : $263\frac{25}{100}$ upon A, the mean square.

As 16 upon C : 4 upon D : : 1.8 upon C : 16.2 upon D, the side of the mean square.

As $20\frac{1}{2}$ upon C : 12 upon D : : 16.2 upon D : $37\frac{5}{12}$ upon C, the answer.

2. The length of a piece of timber is 24.5 feet, and its ends are equal squares, whose sides are each 1.04 feet: what is the solidity? *Ans.* 25 feet 6 inches.

T

3. The length of a piece of timber is 20.38 feet, and the ends are unequal squares, the side of the greater being $19\frac{1}{8}$ inches, and that of the lesser $9\frac{1}{2}$ inches: what is the solidity? *Ans. 29 feet 4 inches.*

4. The length of a piece of timber is 27.36 feet; at the greater end the breadth is 1.78 feet, and the thickness 1.23 feet; and at the lesser end the breadth is 1.04 feet, and the thickness .91 feet: what is the solidity? *Ans. 41.278 feet.*

PROBLEM III.

To find the solidity of round or unsquared timber.

RULE I*.

Multiply the square of the quarter girt (or $\frac{1}{4}$ of the circumference) by the length, and the product will be the content, *according to the common practice.*

* This rule, though commonly used, produces a result about $\frac{1}{4}$ less than the true content of the tree, or nearly what the quantity would be after the tree is hewed square in the usual way: so that it seems intended to make an allowance for the squaring of the tree. When the true solidity is required, use the 2d rule.

When the tree is tapering, the mean girt is found in the same manner as in board measure. Or if the tree be very irregular, the best way is to divide it into a certain number of lengths, and find the content of each part separately.

When trees have their bark on, an allowance is generally made, by deducting so much from the girt as is judged sufficient to reduce it to such a circumference as it would have without its bark. In oak this allowance is about one-tenth or one-twelfth part of the girt; but for elm, beech, ash, &c. whose bark is not so thick, the deduction ought to be less.

By the SLIDING RULE.

As the length upon C : 12 upon D :: $\frac{1}{4}$ girt upon D : content upon C.

EXAMPLES.

1. A piece of timber is $9\frac{3}{4}$ feet long, and the quarter girt is 39 inches: what is the solidity?

By Decimals.

$$3.25 = 39 \text{ inches.}$$

$$3.25$$

$$1625$$

$$650$$

$$975$$

$$10.5625$$

$$9.75 = 9\frac{3}{4} \text{ feet.}$$

$$528125$$

$$739375$$

$$950625$$

$$102.984375 = \text{solidity.}$$

By Cross Multiplication.

fe. in.

$$3 \quad 3 = 39 \text{ inches.}$$

$$3 \quad 3$$

$$9 \quad 9$$

$$9 \quad 9$$

$$10 \quad 6 \quad 9$$

$$9 \quad 9 = 9\frac{3}{4} \text{ feet.}$$

$$95 \quad 0 \quad 9$$

$$7 \quad 11 \quad 0 \quad 9$$

$$102 \quad 11 \quad 9 \quad 9 = \text{solidity.}$$

By the SLIDING RULE.

As $9\frac{3}{4}$ upon C : 12 upon D :: 39 upon D : 103 upon C, the content.

2. The length of a tree is 25 feet, and the girt throughout $2\frac{1}{2}$ feet : what is its solidity ?

Ans. 9 feet 9 inches.

3. The length of a tree is $14\frac{1}{2}$ feet, and its girt in the middle 3.15 feet : required the solidity.

Ans. 9 feet, nearly.

4. The girts of a tree in 4 different places are as follows : in the first place 5 feet 9 inches, in the second 4 feet 5 inches, in the third 4 feet 9 inches, and in the fourth 3 feet 9 inches ; and the length of the whole tree is 15 feet : what is the solidity ?

Ans. 20 feet 5 inches.

5. An oak tree is 45 feet 7 inches long, and its quarter girt 3 feet 8 inches : what is the solid content, allowing $\frac{1}{2}$ for the bark ?

Ans. 515 feet, nearly.

RULE II*.

Multiply the square of $\frac{1}{8}$ of the girt by twice the length, and the product will be the solidity, *extremely near the truth.*

* The following rule was given me by Mr. Burrow, and is a still nearer approximation.

Rule. Multiply the square of the circumference by the length, and take $\frac{1}{11}$ of the product ; from this last number subtract $\frac{1}{8}$ of itself, and the remainder will be the answer.

By the SLIDING RULE.

As twice the length upon C : 12 upon D :: $\frac{1}{8}$ of the girt upon D : content upon C.

EXAMPLES.

1. A piece of timber is $9\frac{3}{4}$ feet long, and $\frac{1}{3}$ of the girt is 2.6 feet : what is the solidity ?

By Decimals.

$$\begin{array}{r}
 2.6 \\
 2.6 \\
 \hline
 156 \\
 52 \\
 \hline
 6.76 \\
 9.75 \\
 \hline
 3380 \\
 4732 \\
 6084 \\
 \hline
 65.9100 \\
 2 \\
 \hline
 131.8200 = \text{content.}
 \end{array}$$

By the SLIDING RULE.

As 19.15 upon C : 12 upon D :: $51\frac{1}{2}$ in. upon D : 132, the content upon C.

2. If the length of a tree be 24 feet, and the girt throughout 8 feet : what is the content ?

Ans. 123 feet, nearly.

3. If a tree girt 14 feet at the thicker end, and 2 feet at the smaller end: required the solidity when the length is 24 feet. *Ans. 123 feet, nearly.*

4. A tree girts in five different places as follows: in the first place 9.43 feet, in the second, 7.92 feet, in the third, 6.15 feet, in the fourth, 4.74 feet, and in the fifth, 3.16 feet; and the whole length is $17\frac{1}{4}$ feet: what is the solidity? *Ans. 54.4249 feet.*

OF

SPECIFIC GRAVITY.

THE specific gravities of bodies are their relative weights contained under the same given magnitude, as a cubic foot, a cubic inch, &c.

The specific gravities of several sorts of bodies are expressed by the numbers annexed to their names in the following table.

A table of the specific gravities of bodies.

Fine gold ———	19640	Brick —————	2000
Standard gold —	18888	Light earth ———	1984
Quicksilver ———	14000	Solid gunpowder —	1745
Lead —————	11325	Sand —————	1520
Fine silver ———	11091	Pitch —————	1150
Standard silver —	10535	Dry box wood ———	1030
Copper —————	9000	Sea water —————	1030
Gun metal ———	8784	Common water ———	1000
Cast brass ———	8000	Dry oak —————	925
Steel —————	7850	Gunpowder, shaken	922
Iron —————	7645	Dry ash —————	800
Cast iron ———	7425	Dry maple ———	755
Tin —————	7320	Dry elm —————	600
Marble —————	2700	Dry fir —————	550
Common stone —	2520	Cork —————	240
Loom —————	2160	Air —————	1 $\frac{1}{4}$

Note. As a cubic foot of water weighs just 1000 ounces Avoirdupois, the numbers in this table ex-

press not only the specific gravities of the several bodies, but also the weight of a cubic foot of each, in Avoirdupois ounces; and hence, by proportion, the weight of any other quantity, or the quantity of any other weight, may be readily known.

PROBLEM I.

To find the magnitude of a body from its weight being given.

RULE.

As the tabular specific gravity of the body, is to its weight in Avoirdupois ounces,

So is one cubic foot, or 1728 cubic inches, to its content in feet, or inches, respectively.

EXAMPLES.

1. Required the content of an irregular block of common stone which weighs 1 cwt. or 112lbs.

$$\begin{array}{r}
 112 \text{ lbs.} \\
 16 \\
 \hline
 672 \\
 112 \\
 \hline
 2520 : 1792 :: 1728 \\
 1728 \\
 \hline
 14336 \\
 3584 \\
 12544 \\
 1792 \\
 \hline
 3096576
 \end{array}$$

2520)3096576(1228 $\frac{4}{5}$ cubic inches the ans.

252

576

504

725

504

2217

2016

2016

2. How many cubic inches of gunpowder, shaken, are there in one pound weight? *Ans.* 30, nearly.

3. How many cubic feet are there in a ton weight of dry oak? *Ans.* 38 $\frac{139}{185}$.

PROBLEM II.

To find the weight of a body from its magnitude being given.

RULE.

As one cubic foot, or 1728 cubic inches, is to the content of the body,

So is its tabular specific gravity, to the weight of the body.

EXAMPLES.

1. Required the weight of a block of marble, whose length is 63 feet, and its breadth and thickness each 12 feet; these being the dimensions of one of the stones in the walls of Balbec.

$$\begin{array}{r}
 63 \\
 12 \\
 \hline
 756 \\
 12 \\
 \hline
 1 : 9072 :: 2700 \\
 2700 \\
 \hline
 6350400 \\
 18144 \\
 \hline
 \end{array}$$

16	{	4		24494400	}	oz.	
		4		6123600			
		112		1530900			lbs.
		20		13668			cwt.

683 ton.

Ans. 683 $\frac{2}{3}$ tons, which is equal to the burthen of an East India ship.

2. What is the weight of a pint of gunpowder ale-measure ? *Ans. 19 oz. nearly.*

3. What is the weight of a block of dry oak, which measures ten feet in length, 3 feet in breadth, and 2 $\frac{1}{4}$ feet deep ? *Ans. 4335 $\frac{5}{8}$ lb.* ✱

PROBLEM III.

To find the specific gravity of a body.

RULE.

Case 1. When the body is heavier than water, weigh it both in water and out of water, and the difference will be the weight lost in the water.

Then, as the weight lost in water, is to the whole weight,

So is the specific gravity of water, to the specific gravity of the body.

EXAMPLES.

A piece of stone weighed in air 10 pounds, but in water only $6\frac{3}{4}$ lbs : required its specific gravity.

$$\begin{array}{r}
 10 \\
 \underline{6\frac{3}{4}} \\
 3\frac{1}{4} : 10 :: 1000 : \\
 13 : 40 :: 1000 : \\
 \hline
 13 \overline{) 40000} (3077 \\
 \underline{39} \\
 100 \\
 \underline{91} \\
 90
 \end{array}$$

Case 2. When the body is lighter than water, so that it will not quite sink ; affix to it another body heavier than water, so that the mass compounded of the two may sink together.

Weigh the heavier body and the compound mass separately both in water and out of it, and find how much each loses in water, by subtracting its weight in water from its weight in air.

Then, as the difference of these remainders is to the weight of the light body in air,

So is the specific gravity of water to the specific gravity of the body.

SPECIFIC GRAVITY.

EXAMPLE.

Suppose a piece of elm weighs in air 15lb. and that a piece of copper which weighs 18lb. in air, and 16lb. in water, is affixed to it, and that the compound weighs 6lb. in water : required the specific gravity of the elm.

	18 in air	39	
	16 in water	6	
	<hr/>	<hr/>	
loss	2	27 loss.	
		2	
		<hr/>	
		25	

• 25 : 15 :: 1000 : 600 Ans.

PROBLEM IV.

To find the quantities of two ingredients in a given compound.

RULE.

Take the differences of every pair of the three specific gravities, *viz.* of the compound and each ingredient, and multiply the difference of every two by the third.

Then as the greatest product is to the whole weight of the compound, so is each of the other products to the weights of the two ingredients.

EXAMPLES.

A composition of 112lb. being made of tin and copper, whose specific gravity is found to be 8784 ;

required the quantity of each ingredient, the specific gravity of tin being 7320, and of copper 9000.

9000	9000	8754
7320	8784	7320
<hr/>	<hr/>	<hr/>
1680	216	1464 <i>diff.</i>
8784	7320	9000
<hr/>	<hr/>	<hr/>
702720	4320	13176000
52704	648	
8784	1512	
<hr/>	<hr/>	
14757120	1581120	

$$14757120 : 112 :: 13176000$$

112

26352000

13176

13176

$$14757120)1475712000(100$$

Ans. 100lb. of copper } *in the composition.*
 and 12lb. of tin }

MISCELLANEOUS QUESTIONS.

1. **W**HAT difference is there between a floor 48 feet long, and 30 feet broad, and two others each of half the dimensions? *Ans. 720 feet.*

2. From a mahogany plank 26 inches broad, a yard and a half is to be sawed off: what distance from the end must the line be struck?

Ans. 6.23 feet.

3. A joist is $8\frac{1}{2}$ inches deep, and $3\frac{1}{2}$ broad: what will be the dimensions of a scantling just as big again as the joist, that is $4\frac{3}{4}$ inches broad?

Ans. 12.52 inches deep.

4. A roof is 24 feet 8 inches by 14 feet 6 inches, and is to be covered with lead at 8*lbs.* to the foot: what will it come to at 18*s.* per cwt.?

*Ans. 22*l.* 19*s.* 10 $\frac{1}{4}$ *d.**

5. What is the side of that equilateral triangle, whose area cost as much paving, at 8*d.* per foot, as the pallisading the three sides did at a guinea per yard?

Ans. 72.746 feet.

6. The two sides of an obtuse-angled triangle are 20 and 40 poles: what must the length of the third side be, that the triangle may contain just an acre?

Ans. 58.876, or 23.099.

7. If two sides of a triangle whose area is $60\sqrt{3}$, be 12 and 20: what is the third side? *Ans. 28.*

8. If an area of 24 be cut off from a triangle, whose three sides are 13, 14, and 15, by a line pa-

MISCELLANEOUS QUESTIONS. 231

parallel to the longest side: what are the lengths of the sides including that area?

Ans. $\frac{13}{7}\sqrt{14}$, $2\sqrt{14}$, and $\frac{15}{7}\sqrt{14}$.

9. The distance of the centres of two circles, whose diameters are each 50, is equal to 30: what is the area of the space inclosed by their circumference?

Ans. 559.119.

10. The area of an equilateral triangle, whose base falls on the diameter, and its vertex in the middle of the arc of a semi-circle, is equal to 100: what is the diameter of the semi-circle?

Ans. 26.3214.

11. The four sides of a field, whose diagonals are equal to each other, are 25, 35, 31, and 19 poles, respectively: what is the area?

Ans. 4 ac. 1 ro. 38 poles.

12. What is the length of a chord which cuts off $\frac{1}{3}$ of the area from a circle whose diameter is 289?

Ans. 278.6716.

13. A cable which is 3 feet long, and 9 inches in compass, weighs 22lbs. what will a fathom of that cable weigh whose diameter is 9 inches?

Ans. 434.26lbs.

14. A circular fish-pond is to be dug in a garden, that shall take up just half an acre: what must the length of the chord be which strikes the circle?

Ans. 27.75 yards.

15. A carpenter is to put an oaken curb to a round well, at 8d. per foot square; the breadth of the curb is to be $7\frac{1}{4}$ inches, and the diameter within $3\frac{1}{2}$ feet: what will be the expence?

Ans. 5s. $2\frac{1}{4}$ d.

16. Suppose the expence of paving a semi-circular plot, at 2s. 4d. per foot, amounted to 10l. what is the diameter of it?

Ans. 14.7737.

* 17. Seven men bought a grinding-stone of 60

* For an excellent geometrical construction of this question, see *Dr. Hutton's Diarian Miscellany*, page 54.

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inches in diameter, each paying $\frac{1}{7}$ part of the expence: what part of the diameter must each grind down for his share?

Ans. The 1st. 4.4508, 2d. 4.8400, 3d. 5.3535, 4th. 6.0765, 5th. 7.2079, 6th. 9.3935, and the 7th. 22.6778.

18. A gentleman has a garden 100 feet long, and 80 feet broad, and a gravel walk is to be made of an equal width half round it: what must the width of the walk be so as to take up just half the ground?

Ans. 25.968 feet.

19. In the midst of a meadow well stored with grass,

I took just an acre to tether my ass;

How long must the cord be, that feeding all round,

He mayn't graze less or more than an acre of ground?

Ans. 39.25073 yards.

20. A maltster has a kiln that is 16 feet 6 inches square; now he wants to pull it down, and build a new one that will dry three times as much at a time as the old one did: what must be the length of its side?

Ans. 28 feet 7 inches.

21. If a round cistern be 26.3 inches diameter, and 52.5 inches deep: how many inches diameter must a cistern be to hold twice the quantity, the depth being the same?

Ans. 37.19 inches.

22. A May-pole, whose top was broken off by a blast of wind, struck the ground at 15 feet distance from the top of the pole: what was the height of the whole May-pole, supposing the length of the broken piece to be 39 feet?

Ans. 75 feet.

23. What will the diameter of a globe be, when the solidity and superficial content thereof are equal to each other?

Ans. 6.

24. How many three inch cubes can be cut out of a 12 inch cube?

Ans. 64.

25. A farmer borrowed part of a hay-rick of his neighbour, which measured 6 feet every way, and

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paid him back again by two equal cubical pieces, each of whose sides were three feet: Query, whether the lender was fully paid?

Ans. He was paid $\frac{1}{4}$ part only.

26. What will the painting a conical church-spire come to at 8*d.* per yard; supposing the circumference of the base to be 64 feet, and the altitude 118 feet?

*Ans. 14*l.* 0*s.* 8 $\frac{3}{4}$ *d.**

27. What will a marble frustum of a cone come to at 12*s.* per solid foot; the diameter of the greater end being 4 feet, that of the lesser end $1\frac{1}{2}$ feet, and the length of the slant side 8 feet?

*Ans. 30*l.* 1*s.* 10*d.**

28. The diameter of a legal Winchester bushel is $18\frac{1}{2}$ inches, and its depth 8 inches: what must the diameter of that bushel be whose depth is $7\frac{1}{2}$ inches?

Ans. 19.10671.

29. Three men bought a tapering piece of timber, which was the frustum of a square pyramid; one side of the greater end was 3 feet, one side of the lesser end 1 foot, and the length 18 feet: what is the length of each man's piece, supposing they paid equally, and are to have equal shares? *Ans. 1*st.* 3.269, 2*d.* 4.559, and the 3*d.* 10.172, reckoning from the greater end to the less.*

30. Suppose the ball at the top of St. Paul's Church is 6 feet in diameter: what did the gilding of it come to at $3\frac{1}{2}$ *d.* per square inch? *Ans. 237*l.* 1*s.* 10 $\frac{1}{4}$ *d.**

31. A person wants a cylindric vessel of 3 feet deep, that shall hold twice as much as a vessel of 28 inches deep, and 46 inches in diameter: what must be the diameter of the vessel required?

Ans. 57.37 inches.

32. Two porters agreed to drink off a quart of strong beer between them, at two pulls, or a draught each; now, the first having given it a black eye, as it is called, or drank till the surface of the liquor touched the opposite edge of the bottom, gave the

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remaining part of it to the other: what was the difference of their shares, supposing the pot was the frustum of a cone, the depth being 5.7 inches, the diameter at the top 3.7 inches, and that of the bottom 4.23 inches? *Ans. 7.07 cubic inches.*

33. Three persons having bought a sugar-loaf, want to divide it equally amongst them by sections parallel to the base; it is required to find the altitude of each person's share, supposing the loaf to be a cone, whose height is 20 inches. *Ans. 13.867 the upper part, 3.604 the middle part, and 2.528 the lower part.*

34. How high above the surface of the earth must a person be raised to see $\frac{1}{3}$ of its surface?

Ans. To the height of the earth's diameter.

35. A cubical foot of brass is to be drawn into a wire of $\frac{1}{40}$ of an inch in diameter: what will be the length of the wire, allowing no loss in the metal?

Ans. 97784.797 yards, or near 56 miles.

36. A gentleman has a bowling-green, 300 feet long, and 200 feet broad, which he would raise one foot higher, by means of the earth to be dug out of a ditch that goes round it: to what depth must the ditch be dug, supposing its breadth to be every where 8 feet?

Ans. $7\frac{13}{286}$ feet.

37. Of what diameter must the bore of a cannon be, which is cast for a ball of 24lbs. weight, so that the diameter of the bore may be $\frac{1}{10}$ of an inch more than that of the ball?

Ans. 5.757 inches.

38. The ellipse in Grosvenor square measures 840 links across the longest way, and 612 the shortest, within the rails: now the walls being 14 inches thick, it is required to find what ground they inclose, and what they stand upon? *Ans. They inclose 4 ac. 0 ro. 6 po. and stand on $1760\frac{1}{2}$ square feet.*

39. If a heavy sphere whose diameter is 4 inches, be put into a conical glass, full of water, whose dia-

meter is 5, and altitude 6 inches; it is required to find how much water will run over?

Ans. $\frac{35}{47}$ of a pint nearly.

40. Suppose it be found, by measurement, that a man of war, with its ordnance, rigging, and appointments, draws so much water as to displace 50000 cubic feet of water: required the weight of the vessel.

Ans. $1395\frac{1}{16}$ tons.

41. One ev'ning I chanc'd with a tinker to sit,
Whose tongue ran a great deal too fast for his wit;
He talk'd of his art with abundance of mettle;
So I ask'd him to make me a flat-bottom'd kettle:
Let the top and the bottom diameters be,
In just such proportion as five is to three:
Twelve inches the depth I propos'd, and no more;
And to hold in ale gallons seven less than a score.
He promis'd to do it, and straight to work went;
But when he had done he found it too scant.
He altered it then, but too big he had made it;
For though it held right, the diameters fail'd it:
Thus making it often too big and too little,
The Tinker at last had quite spoilt his kettle;
But he says he will bring his said promise to pass,
Or else that he'll spoil every ounce of his brass.
Now to keep him from ruin, I pray find him out
The diameter's length, for he'll ne'er do it I doubt.

Ans. The bottom diameter is 14.64017, and the top diameter 24.40028.

A
TABLE

OF THE

AREAS OF THE SEGMENTS OF A CIRCLE,

Whose diameter is unity, and supposed to be divided
into 1000 equal parts.

Ver- sed Sine.	Seg. Area.	Ver- sed Sine.	Seg. Area.	Ver- sed Sine.	Seg. Area.
.001	.000042	.024	.004921	.047	.013392
.002	.000119	.025	.005230	.048	.013818
.003	.000219	.026	.005546	.049	.014247
.004	.000337	.027	.005867	.050	.014681
.005	.000470	.028	.006194	.051	.015119
.006	.000618	.029	.006527	.052	.015561
.007	.000779	.030	.006865	.053	.016007
.008	.000951	.031	.007209	.054	.016457
.009	.001135	.032	.007558	.055	.016911
.010	.001329	.033	.007913	.056	.017369
.011	.001533	.034	.008273	.057	.017831
.012	.001746	.035	.008638	.058	.018296
.013	.001963	.036	.009008	.059	.018766
.014	.002199	.037	.009383	.060	.019239
.015	.002438	.038	.009763	.061	.019716
.016	.002685	.039	.010148	.062	.020196
.017	.002940	.040	.010537	.063	.020680
.018	.003202	.041	.010931	.064	.021168
.019	.003471	.042	.011330	.065	.021659
.020	.003748	.043	.011734	.066	.022154
.021	.004031	.044	.012142	.067	.022652
.022	.004322	.045	.012554	.068	.023154
.023	.004618	.046	.012971	.069	.023659

The Areas of the Segments of a Circle.

Ver- sed Sine.	Seg. Area.	Ver- sed Sine.	Seg. Area.	Ver- sed Sine.	Seg. Area.
.070	.024168	.103	.042687	.136	.064074
.071	.024680	.104	.043296	.137	.064760
.072	.025195	.105	.043908	.138	.065449
.073	.025714	.106	.044522	.139	.066140
.074	.026236	.107	.045139	.140	.066833
.075	.026761	.108	.045759	.141	.067528
.076	.027289	.109	.046381	.142	.068225
.077	.027821	.110	.047005	.143	.068924
.078	.028356	.111	.047632	.144	.069625
.079	.028894	.112	.048262	.145	.070328
.080	.029435	.113	.048894	.146	.071033
.081	.029979	.114	.049528	.147	.071741
.082	.030526	.115	.050165	.148	.072450
.083	.031076	.116	.050804	.149	.073161
.084	.031629	.117	.051446	.150	.073874
.085	.032186	.118	.052090	.151	.074589
.086	.032745	.119	.052736	.152	.075306
.087	.033307	.120	.053385	.153	.076026
.088	.033872	.121	.054036	.154	.076747
.089	.034441	.122	.054689	.155	.077469
.090	.035011	.123	.055345	.156	.078194
.091	.035585	.124	.056003	.157	.078921
.092	.036162	.125	.056663	.158	.079649
.093	.036741	.126	.057326	.159	.080380
.094	.037323	.127	.057991	.160	.081112
.095	.037909	.128	.058658	.161	.081846
.096	.038496	.129	.059327	.162	.082582
.097	.039087	.130	.059999	.163	.083320
.098	.039680	.131	.060672	.164	.084059
.099	.040276	.132	.061348	.165	.084801
.100	.040875	.133	.062026	.166	.085544
.101	.041476	.134	.062707	.167	.086289
.102	.042080	.135	.063389	.168	.087036

The Areas of the Segments of a Circle.

Ver- sed Sine.	<i>Seg. Area.</i>	Ver- sed Sine.	<i>Seg. Area.</i>	Ver- sed Sine.	<i>Seg. Area.</i>
.169	.087785	.202	.113426	.235	.140688
.170	.088535	.203	.114230	.236	.141537
.171	.089287	.204	.115035	.237	.142387
.172	.090041	.205	.115842	.238	.143238
.173	.090797	.206	.116650	.239	.144091
.174	.091554	.207	.117460	.240	.144944
.175	.092313	.208	.118271	.241	.145799
.176	.093074	.209	.119083	.242	.146655
.177	.093836	.210	.119897	.243	.147512
.178	.094601	.211	.120712	.244	.148371
.179	.095366	.212	.121529	.245	.149230
.180	.096134	.213	.122347	.246	.150091
.181	.096903	.214	.123167	.247	.150953
.182	.097674	.215	.123988	.248	.151816
.183	.098447	.216	.124810	.249	.152680
.184	.099221	.217	.125634	.250	.153546
.185	.099997	.218	.126459	.251	.154412
.186	.100774	.219	.127285	.252	.155280
.187	.101553	.220	.128113	.253	.156149
.188	.102334	.221	.128942	.254	.157019
.189	.103116	.222	.129773	.255	.157890
.190	.103900	.223	.130605	.256	.158762
.191	.104685	.224	.131438	.257	.159636
.192	.105472	.225	.132272	.258	.160510
.193	.106261	.226	.133108	.259	.161386
.194	.107051	.227	.133945	.260	.162263
.195	.107842	.228	.134784	.261	.163140
.196	.108636	.229	.135624	.262	.164019
.197	.109430	.230	.136465	.263	.164899
.198	.110226	.231	.137307	.264	.165780
.199	.111024	.232	.138150	.265	.166663
.200	.111823	.233	.138995	.266	.167546
.201	.112624	.234	.139841	.267	.168430

The Areas of the Segments of a Circle.

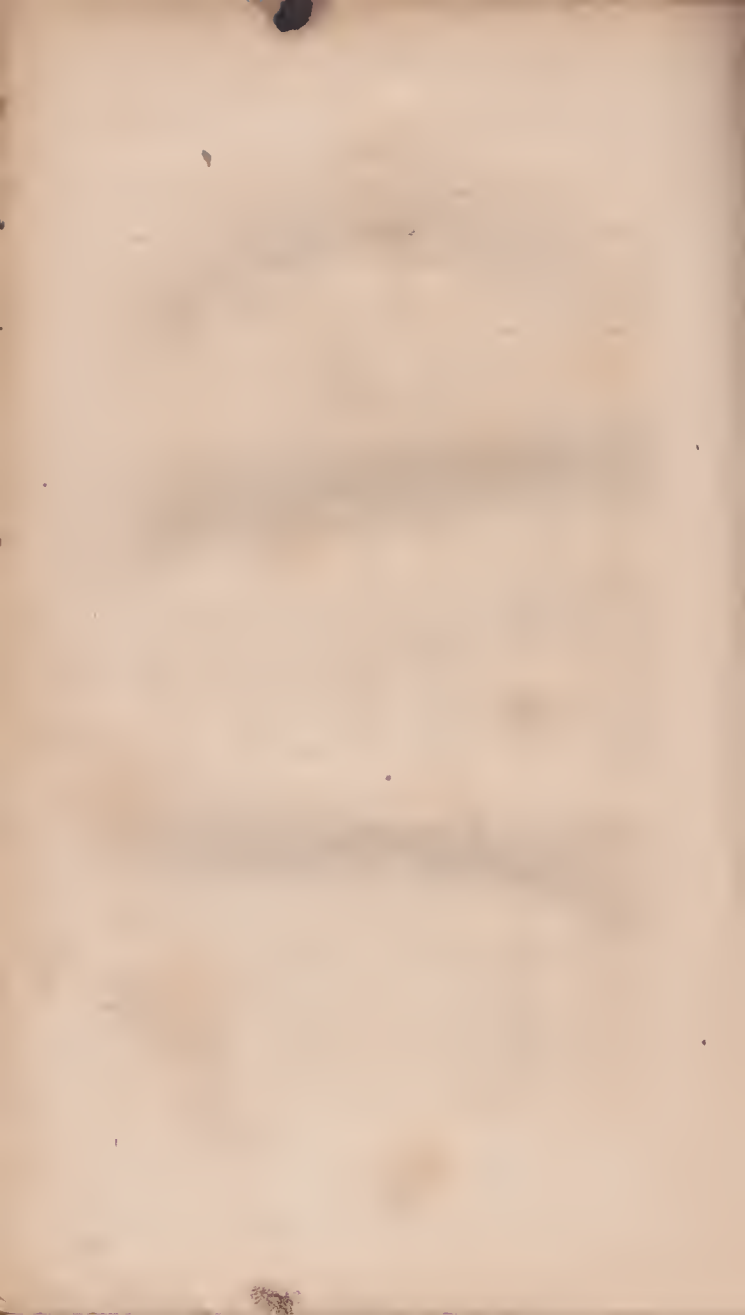
Ver- sed Sine.	<i>Seg. Area.</i>	Ver- sed Sine.	<i>Seg. Area.</i>	Ver- sed Sine.	<i>Seg. Area.</i>
.268	.169315	.301	.199085	.334	.229801
.269	.170202	.302	.200003	.335	.230745
.270	.171089	.303	.200922	.336	.231689
.271	.171978	.304	.201841	.337	.232634
.272	.172867	.305	.202761	.338	.233580
.273	.173758	.306	.203683	.339	.234526
.274	.174649	.307	.204605	.340	.235473
.275	.175542	.308	.205527	.341	.236421
.276	.176435	.309	.206451	.342	.237369
.277	.177330	.310	.207376	.343	.238318
.278	.178225	.311	.208301	.344	.239268
.279	.179122	.312	.209227	.345	.240218
.280	.180019	.313	.210154	.346	.241169
.281	.180918	.314	.211082	.347	.242121
.282	.181817	.315	.212011	.348	.243074
.283	.182718	.316	.212940	.349	.244026
.284	.183619	.317	.213871	.350	.244980
.285	.184521	.318	.214802	.351	.245934
.286	.185425	.319	.215733	.352	.246889
.287	.186329	.320	.216666	.353	.247845
.288	.187234	.321	.217599	.354	.248801
.289	.188140	.322	.218533	.355	.249757
.290	.189047	.323	.219468	.356	.250715
.291	.189955	.324	.220404	.357	.251673
.292	.190864	.325	.221340	.358	.252631
.293	.191775	.326	.222277	.359	.253590
.294	.192684	.327	.223215	.360	.254550
.295	.192596	.328	.224154	.361	.255510
.296	.194509	.329	.225093	.362	.256471
.297	.195422	.330	.226033	.363	.257433
.298	.196337	.331	.226974	.364	.258395
.299	.197252	.332	.227915	.365	.259357
.300	.198168	.333	.228858	.366	.260320

The Areas of the Segments of a Circle.

Ver- sed Sine.	Seg. Area.	Ver- sed Sine.	Seg. Area.	Ver- sed Sine.	Seg. Area.
.367	.261284	.400	.293369	.433	.325900
.368	.262248	.401	.294349	.434	.326892
.369	.263213	.402	.295330	.435	.327882
.370	.264178	.403	.296311	.436	.328874
.371	.265144	.404	.297292	.437	.329866
.372	.266111	.405	.298273	.438	.330858
.373	.267078	.406	.299255	.439	.331850
.374	.268045	.407	.300238	.440	.332843
.375	.269013	.408	.301220	.441	.333836
.376	.269982	.409	.302203	.442	.334829
.377	.270951	.410	.303187	.443	.335822
.378	.271920	.411	.304171	.444	.336816
.379	.272890	.412	.305155	.445	.337810
.380	.273861	.413	.306140	.446	.338804
.381	.274832	.414	.307125	.447	.339798
.382	.275803	.415	.308110	.448	.340793
.383	.276775	.416	.309095	.449	.341787
.384	.277748	.417	.310081	.450	.342782
.385	.278721	.418	.311068	.451	.343777
.386	.279694	.419	.312054	.452	.344772
.387	.280668	.420	.313041	.453	.345768
.388	.281642	.421	.314029	.454	.346764
.389	.282617	.422	.315016	.455	.347759
.390	.283592	.423	.316004	.456	.348755
.391	.284568	.424	.316992	.457	.349752
.392	.285544	.425	.317981	.458	.350748
.393	.286521	.426	.318970	.459	.351745
.394	.287498	.427	.319959	.460	.352741
.395	.288476	.428	.320948	.461	.353739
.396	.289453	.429	.321938	.462	.354736
.397	.290432	.430	.322928	.463	.355732
.398	.291411	.431	.323918	.464	.356730
.399	.292390	.432	.324909	.465	.357727

The Areas of the Segments of a Circle.

Ver- sed Sine.	<i>Seg. Area.</i>	Ver- sed Sine.	<i>Seg. Area.</i>	Ver- sed Sine.	<i>Seg. Area.</i>
.466	.358725	.478	.370706	.490	.382699
.467	.359723	.479	.371705	.491	.383699
.468	.360721	.480	.372764	.492	.384699
.469	.361719	.481	.373703	.493	.385699
.470	.362717	.482	.374702	.494	.386699
.471	.363715	.483	.375702	.495	.387699
.472	.364713	.484	.376702	.496	.388699
.473	.365712	.485	.377701	.497	.389699
.474	.366710	.486	.378701	.498	.390699
.475	.367709	.487	.379700	.499	.391699
.476	.368708	.488	.380700	.500	.392699
.477	.369707	.489	.381699		



APPENDIX.



OF

GAUGING.

THE business of cask-gauging is commonly performed by two instruments, namely, the gauging or sliding rule, and the gauging or diagonal rod.

I. OF THE GAUGING RULE.

This instrument serves to compute the contents of casks, &c. after the dimensions have been taken. It is a square rule, having various logarithmic lines on its four sides or faces; and three sliding pieces, running in grooves, in three of them.

Upon the first face are three lines, namely, two marked A, B, for multiplying and dividing; and the third, MD, for malt depth, because it serves to gauge malt. The middle one B is on the slider, and is a kind of double line, being marked at both the edges of the slider, for applying it to both the lines A and MD. These three lines are all of the same radius or distance from one to 10, each containing twice the

length of the radius. A and B are placed and numbered exactly alike, each beginning at 1, which may be either 1, or 10, or 100, &c. or .1, or .01, or .001, &c. but whatever it is, the middle division, 10, will be ten times as much, and the last division 100 times as much. But 1 on the line MD is opposite 215, or more exactly 2150.4 on the other lines, which number 2150.4 denotes the cubic inches in a malt bushel; and its divisions numbered retrograde to those of A and B. On these two lines are also several other marks and letters: thus, on the line A are MB, for malt bushel, at the number 2150.4; and A for ale, at 282, the cubic inches in an ale gallon; and on the line B, is W, for wine, at 231, the cubic inches in a wine gallon; also, *si*, for square inscribed, at .707, the side of a square inscribed in a circle whose diameter is 1; *se*, for square equal, at .886, the side of a square which is equal to the same circle; and *c*, for circumference, at 3.1416, the circumference of the same circle.

On the second face, or that opposite the first, are a slider and four lines, marked D, C, D, E, at one end, and root, square, root, cube, at the other; the lines C and D containing respectively the squares and cubes of the opposite numbers on the lines D, D; the radius of D being double to that of A, B, C, and triple to that of E; so that whatever the first 1 on D denotes, the first on C is the square of it, and the first on E the cube of it; so if D begin with 1, C and E will begin with 1; but if D begin with 10, C will begin with 100, and E with 1000; and so on. On the line C are marked *oc* at .0796, for the area of the circle, whose circumference is 1; and *od* at .7854, for the area of the circle whose diameter is 1. Also on the line D, are WG, for wine guage, at 17.15; and AG for ale guage, at 18.95; and MR, for malt round, at 52.32; these three being

the gauge points for round and circular measure, and are found by dividing the square roots of 231, 282, and 2150.4, by the square root of .7854: also MS, for malt square, are marked at 46.37, the malt gauge point for square measure, being the square root of 2150.4.

On the third face are three lines, one on a slider marked N; and two on the stock, marked SS and SL, for segment standing and segment lying, which serve for ullaging standing and lying casks.

And on the fourth, or opposite face, are a scale of inches, and three other scales, marked spheroid, or 1st variety, 2d variety, 3d variety; the scale for the fourth or conic variety, being on the inside of the slider in the third face. The use of these lines is to find the mean diameters of casks.

Besides all those lines, there are two others on the insides of the two first sliders, being continued from the one slider to the other. The one of these is a scale of inches, from $12\frac{1}{2}$ to 36; and the other is a scale of ale gallons, between the corresponding numbers 435 and 3.61; which form a table to show, in ale gallons, the contents of all cylinders whose diameters are from $12\frac{1}{2}$ to 36 inches, their common altitude being 1 inch.

The use of the Gauging Rule.

PROBLEM I.

To multiply two numbers, as 12 and 25.

Set 1 on B, to either of the given numbers, as 12, on A; then against 25 on B, stands 300 on A; which is the product.

X 2

PROBLEM II.

To divide one number by another, as 300 by 25.

Set 1 on B, to 25 on A; then against 300 on A, stands 12 on B, for the quotient.

PROBLEM III.

To find a fourth proportional, as 8, 24, and 96.

Set 8 on B, to 24 on A; then against 96 on B, is 288 on A, the 4th proportional to 8, 24, 96, required.

PROBLEM IV.

To extract the square root, as of 225.

The first 1 on C standing opposite the one on D, on the stock; then against 225 on C, stands its square root 15 on D.

PROBLEM V.

To extract the cube root, as of 3375.

The line D on the slide being set straight with E; then opposite 3375 on E stands its cube root 15 on D.

PROBLEM VI.

To find a mean proportional, as between 4 and 9.

Set 4 on C, to the same 4 on D; then against 9 on C, stands the mean-proportional 6 on D.

PROBLEM VII.

To find numbers in duplicate proportion.

As, to find a number which shall be to 120 as the square of 3 to the square of 2.

Set 2 on D, to 120 on C; then against 3 on D, stands 270 on C, for the answer.

PROBLEM VIII.

To find numbers in subduplicate proportion.

As, to find a number which shall be to 2 as the root of 270 to the root of 120.

Set 2 on D, to 120 on C; then against 270 on C, stands 3 on D, for the answer.

PROBLEM IX.

To find numbers in triplicate proportion.

As, to find a number which shall be to 100, as the cube of 36 is to the cube of 40.

Set 40 on D, to 100 on E; then against 36 on D, stands 72.9 on E, for the answer.

PROBLEM X.

To find numbers in subtriplicate proportion.

As, to find a number which shall be to 40, as the cube root of 72.9 is to the cube root of 100.

Set 40 on D, to 100 on E; then against 72.9 on E, stands 36 on D, for the answer.

PROBLEM XI.

To compute malt bushels by the line MD.

As, to find the malt bushels in the couch, floor, or cistern, whose length is 230, breadth 58.2, and depth 5.4 inches

Set 230 on B, to 5.4 on MD; then against 58.2 on A, stands 33.6 bushels on B, for the answer.

Note. The uses of the other marks on the rule, will appear in the examples farther on.

OF THE GAUGING OR DIAGONAL ROD.

The diagonal rod is a square rule, having four faces; being commonly four feet long, and folding together by joints. This instrument is used both for gauging or measuring casks, and computing their contents, and that from one dimension only, namely the diagonal of the cask, or the length from the middle of the bung hole to the meeting of the head of the cask with the stave opposite to the bung; being the longest straight line that can be drawn within the cask from the middle of the bung. And, accordingly, on one face of the rule is a scale of inches for

measuring this diagonal; to which are placed the areas, in ale gallons, of circles to the corresponding diameters, in like manner as the lines on the under sides of the three slides in the sliding rule.

On the opposite face, are two scales of ale and wine gallons, expressing the contents of casks having the corresponding diagonals. And these are the lines which chiefly form the difference between this instrument and the sliding rule; for all their other lines are the same, and are to be used in the same manner.

EXAMPLE.

The rod being applied within the cask at the bung hole, the diagonal was found to be 34.4 inches; required the content in gallons.

Now to 34.4 inches correspond, on the rod, $90\frac{3}{4}$ ale gallons, or 111 wine gallons, the content required.

Note. The contents exhibited by the rod, answer to the most common form of casks, and fall in between the 2d and 3d varieties following.

OF CASKS AS DIVIDED INTO VARIETIES.

It is usual to divide casks into four cases or varieties, which are judged of from the greater or less apparent curvature of their sides; namely,

1. The middle frustum of a spheroid,
2. The middle frustum of a parabolic spindle,
3. The two equal frustums of a paraboloid,
4. The two equal frustums of a cone.

And if the content of any of these be computed in inches, by their proper rules, and this be divided by 282, or 231, or 2150.4, the quotient will be the content in ale gallons, or wine gallons, or malt bushels, respectively. Because

282	cubic inches make	1 ale gallon
231	-	1 wine gallon
2150.4	-	1 malt bushel.

And the particular rule will be for each as in the following problems :

PROBLEM XII.

To find the content of a cask of the first form.

To the square of the head diameter, add double the square of the bung diameter; and multiply the sum by the length of the cask. Then let the product

be multiplied by $.0009\frac{1}{4}$, or divided by 1077, for ale gallons;

and multiplied by $.0011\frac{1}{3}$, or divided by 882 for wine gallons.

EXAMPLES.

1. Required the content of a spheroidal cask, whose length is 40, and bung and head diameters 32 and 24 inches.

24	32
24	32
—	—
96	64
48	96
—	—
576	1024
—	—
	2



2048	104960	104960
576	$.0009\frac{1}{4}$	$.0011\frac{1}{3}$
—	—	—
2624	944640	1154560
40	26240	34987
—	—	—
104960	ale	97.0880 gallons
		118.9547 wine.

By the Gauging Rule.

Having set 40 on C, to the ale gauge 32.82 on D,
against

24 on D, stands 21.3 on C

32 on D, stands 38.0 on C

the same 38.0

sum 97.3 ale gallons.

And having set 40 on C, to the wine gauge 29.7
on D,

against

24 on D, stands 26.1 on C

32 on D, stands 46.5 on C

the same 46.5

sum 119.1 wine gallons.

Ex. 2. Required the content of the spheroidal
cask, whose length is 20, and diameters 12 and 16
inches.

Answer $\left\{ \begin{array}{l} 12.136 \text{ ale gal'ns,} \\ 14.869 \text{ wine gallons.} \end{array} \right.$

PROBLEM XIII.

To find the content of a cask of the second form.

To the square of the head diameter, add double
the square of the bung diameter, and from the sum
take $\frac{2}{3}$ or $\frac{4}{10}$ of the square of the difference of the
diameters; then multiply the remainder by the
length, and the product again by .0009 $\frac{1}{4}$ for ale
gallons, or by .0011 $\frac{1}{4}$ for wine gallons.

EXAMPLES.

1. The length being 40, and diameters 24 and 32, required the content.



32			
24			
<hr/>			
8	2624.0	103936	103936
8	25 6	.0009 $\frac{1}{4}$.0011 $\frac{1}{5}$
<hr/>			
64	2598.4	935424	1143296
4	40	25984	34645
<hr/>			
25.6	103936	ale 96.1408	gall. 117.9741 wine.
<hr/>			

By the Gauging Rule.

Having set 40 on C, to 32.82 on D, against 8 on D, stands 2.4 on C; the $\frac{4}{10}$ of which is 0.96. This taken from the 97.3 in the last form, leaves 96.3 ale gallons.

And having set 40 on C, to 29.7 on D, against 8 on D, stands 2.9 on C; the $\frac{4}{10}$ of which is 1.16. This taken from the 119.1 in the last form leaves 117.9 wine gallons.

Ex. 2. Required the content when the length is 20, and the diameters 12 and 16.

Answer $\left\{ \begin{array}{l} 12.018 \text{ ale gallons,} \\ 14.724 \text{ wine gallons.} \end{array} \right.$

PROBLEM XIV.

To find the content of a cask of the third form.

To the square of the bung diameter add the square of the head diameter; multiply the sum by the length, and the product again by .0014 for ale gallons, or by .0017 for wine gallons.

EXAMPLES.

1. Required the content of a cask of the third form, when the length is 40, and the diameters 24 and 32.



1024	64000	64000
576	.0014	.0017
<hr/>	<hr/>	<hr/>
1600	256	448
40	64	64
<hr/>	<hr/>	<hr/>
64000	ale 89.6 gallons.	108.8 wine.

By the Gauging Rule.

Set 40 on C, to 26.8 on D; then against
 24 on D, stands 32.0 on C
 32 on D, stands 57.3 on C

sum 89.3 ale galls.

And having set 40 on C, to 24.25 on D ; then
 against 24 on D, stands 39.1 on C
 32 on D, stands 69.8 on C

sum 108.9 wine gallons.

Ex. 2. Required the content when the length is 20, and the diameters 12 and 16.

Answer $\left\{ \begin{array}{l} 11.2 \text{ ale gallons,} \\ 13.6 \text{ wine gallons.} \end{array} \right.$

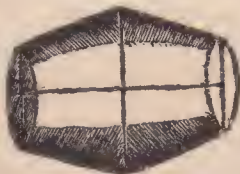
PROBLEM XV.

To find the content of a cask of the fourth form.

Add the square of the difference of the diameters to 3 times the square of their sum; then multiply the sum by the length, and the product again by $.00023\frac{1}{2}$ for ale gallons, or by $.00028\frac{1}{3}$ for wine gallons.

EXAMPLES.

1. Required the content, when the length is 40, and the diameters 24 and 32 inches.



56	8		
56	8		
<hr/>	<hr/>	378880	378880
336	64	.00023 $\frac{1}{3}$.00028 $\frac{1}{3}$
280	9408	<hr/>	<hr/>
<hr/>	<hr/>	1136640	5031040
3136	9472	757760	757760
3	40	75776	126293
<hr/>	<hr/>	<hr/>	<hr/>
9408	378880	ale 87.90016 gall. 107.34933 wine.	

By the Sliding Rule.

Set 40 on C, to 65.64 on D; then against

8 on D, stands 0.6 on C

56 on D, stands 29.1 on C

29.1

29.1

sum 87.9 ale gallons.

And set 40 on C, to 59.41 on D; then against

8 on D, stands 0.7

56 on D, stands 35.6

35.6

35.6

sum 107.5 wine gall.

Ex. 2. What is the content of a conical cask, the length being 20, and the bung and head diameters 16 and 12 inches?

Answer { 10.985 ale gallons,
 { 13.416 wine gallons.

PROBLEM XVI.

To find the content of a cask by four dimensions.

Add together the squares of the bung and head diameters, and the square of double the diameter taken in the middle between the bung and head; then multiply the sum by the length of the cask, and the product again by $.0004\frac{2}{3}$ for ale gallons, or by $.0005\frac{2}{3}$ for wine gallons.

EXAMPLES.

1. Required the content of any cask whose length is 40, the bung diameter being 32, the head diameter 24, and the middle diameter between the bung and the head $28\frac{3}{4}$ inches.

57.5	24	32
57.5	24	32
<hr/>	<hr/>	<hr/>
2875	96	64
4025	48	96
2875	<hr/>	<hr/>
<hr/>	576	1024
3306.25	<hr/>	<hr/>
1024		
576		
<hr/>		
4906.25		
40		
<hr/>		
196250	196250	
$.0004\frac{2}{3}$	$.0005\frac{2}{3}$	
<hr/>	<hr/>	
785000	981250	
130833	130833	
<hr/>	<hr/>	

ale 91.5833 gallons 111.2083 wine.

By the Sliding Rule.

Set 40 on C, to 46.4 on D; then against
 24 on D, stands 10.5
 32 on D, stands 19.0
 57 $\frac{1}{2}$ on D, stands 62.0

sum 91.5 ale gallons.

Set 40 on C, to 24.0 on D; then against
 24 on D, stands 13.0
 32 on D, stands 23.2
 57 $\frac{1}{2}$ on D, stands 75.0

sum 111.2 wine gallons.

Ex. 2. What is the content of a cask, whose length is 20, the bung diameter being 16, the head diameter 12, and the diameter in the middle between them 14 $\frac{3}{8}$?

Answer $\left\{ \begin{array}{l} 11.4479 \text{ ale gallons,} \\ 13.9010 \text{ wine gallons.} \end{array} \right.$

PROBLEM XVII.

To find the content of any cask from three dimensions only.

Add into one sum 39 times the square of the bung diameter, 25 times the square of the head diameter, and 26 times the product of the two diameters: then multiply the sum by the length, and the product again by $\frac{.00034}{9}$ for wine gallons, or by $\frac{.00034}{11}$, or .00003 $\frac{1}{11}$, for ale gallons.

Y 2

EXAMPLES.

1. Required the content of a cask, whose length is 40, and the bung and head diameters 32 and 24.

32	24	32
32	24	24
<hr/>	<hr/>	<hr/>
64	96	128
96	48	64
<hr/>	<hr/>	<hr/>
1024	576	768
39	25	26
<hr/>	<hr/>	<hr/>
9216	2880	4608
3072	1152	1536
<hr/>	<hr/>	<hr/>
39936	14400	19968
<hr/>	<hr/>	<hr/>
	39936	
	19968	
	<hr/>	
	74304	
	40	
	<hr/>	
	2972160	
	.00034	2972160
	<hr/>	.00003 $\frac{1}{11}$
	11888640	<hr/>
	8916480	8916480
	<hr/>	270196
	<hr/>	<hr/>
9)1010.53440		
	112.2816 wine gal.	91.86676 ale gal.

Ex. 2. What is the content of a cask, whose length is 20, and the bung and head diameters 16 and 12?

Answer: $\begin{cases} 11.4833 \text{ ale gallons,} \\ 14.0352 \text{ wine gallons.} \end{cases}$

Note. This is the most exact rule of any, for three dimensions only; and agrees nearly with the diagonal rod.

OF THE ULLAGE OF CASKS.

The ullage of a cask is what it contains when only partly filled. And it is considered in two positions, namely, as standing on its end with the axes perpendicular to the horizon, or as lying on its side with the axes parallel to the horizon.

PROBLEM XVIII.

To find the ullage by the Sliding Rule.

By one of the preceding problems find the whole content of the cask. Then set the length on N, to 100 on SS, for a segment standing, or set the bung diameter on N, to 100 on SL, for a segment lying; then against the wet inches on N, is a number on SS or SL, to be reserved.

Next, set 100 on B, to the reserved number on A; then against the whole content on B, will be found the ullage on A.

EXAMPLES:

1. Required the ullage answering to 10 wet inches of a standing cask, the whole content of which is 92 gallons, and length 40 inches:

Having set 40 on N, to 100 on SS; then against 10 on N, is. 23 on SS, the reserved number.

Then set 100 on B, to 23 on A; and against 92 on B, is 21.2 on A, the ullage required.

Ex. 2. What is the ullage of a standing cask, whose whole length is 20 inches, and content $11\frac{1}{2}$ gallons; the wet inches being 5? Ans. 2.65. galls.

Ex. 3. The content of a cask being 92 gallons, and the bung diameter 32, required the ullage of the segment lying when the wet inches are 8.

Ans. 16.4 gallons.

PROBLEM XIX.

To ullage a standing cask by the pen.

Add all together the square of the diameter at the surface of the liquor, the square of the diameter of the nearest end, and the square of double the diameter taken in the middle between the other two; then multiply the sum by the length between the surface and nearest end, and the product again by $.0004\frac{2}{3}$ for ale gallons, or by $.0005\frac{2}{3}$ for wine gallons, in the less part of the cask, whether empty or filled.

EXAMPLES.

The three diameters being 24, 27, and 29 inches, required the ullage for 10 wet inches.

24	29	54	
24	29	54	2916
—	—	—	841
96	261	216	576
48	58	270	—
—	—	—	4333
576	841	2916	10
—	—	—	—
	43330		43330
	.0004 $\frac{2}{3}$.0005 $\frac{2}{3}$
	—		—
	173320		216650
	28885		28885
	—		—
ale	20.2205 gallons		24.5535 wine.
	—		—

PROBLEM XX.

To ullage a lying cask by the pen.

Divide the wet inches by the bung diameter, find the quotient in the column of versed sines, in the table of circular segments at page 236 of the book, taking out its corresponding segment. Then multiply this segment by the whole content of the cask, and the product again by $1\frac{1}{4}$ for the ullage required, nearly.

EXAMPLES.

Supposing the bung diameter 32, and content 92 ale gallons; to find the ullage for 8 wet inches.

32)8(.25, whose tab. seg. is .153546

92

307092

1381914

14.126232

$\frac{1}{4}$ is 3.531558

17.657790 Answer.

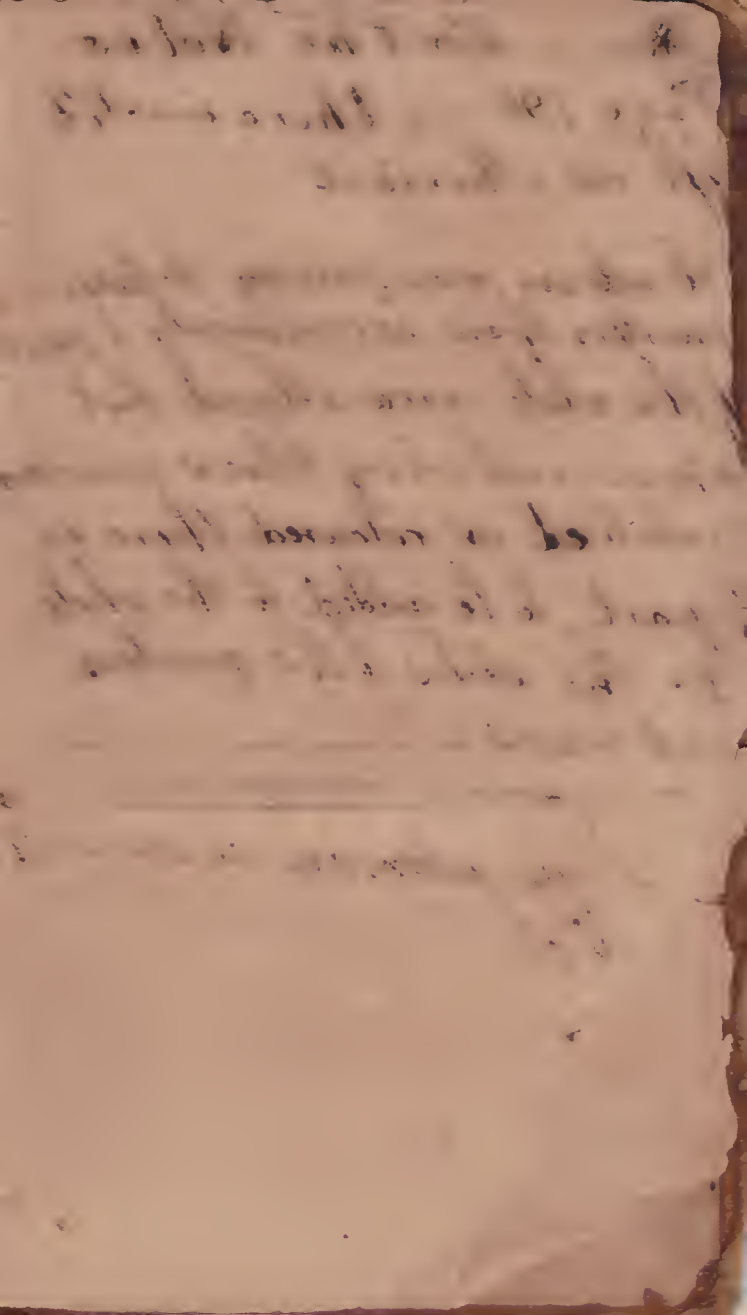
THE END.

FINIS











For the Notes

Page 190 = There is a Note
left out, this is it

And in measuring between
quarters there is commonly $\frac{1}{5}$ part
of the whole area allowed but
when rendering between quarters
is whitened or coloured there is
 $\frac{1}{5}$ part to be added to the whole
for the sides of the quarters
and braces

Nine gallons are in use in the
R. F.

Nid

11th Londoner

The Answer in page 146. 2

Sum is 1795. 45 54

First sum in Prob. 22 the
answer is 659.756.

Page 184 = 8" 19" 9 $\frac{1}{2}$ m *

187 = 12" 11" 6 *

189 = 65" 24" 6 *

191 = 176.875 *

194 = 4" 16" 3 $\frac{1}{4}$ *

226 = 4335.74 *

W Penington

W Penington 1888

William Penington

W

